TH-PT PreCalculus - Trigonometry Session 2

$6^{\frac{1}{2}} = \sqrt{6}$

$81^{3/2} = \left(\sqrt{81}\right)^3 = \pm 729$

$9^3 = 729$ $\quad -9^3 = -729$

$36^{k-3} = 216^{3k}$

$(6^2)^{k-3} = (6^3)^{3k}$

$6^{2k-6} = 6^{9k}$

$2k - 6 = 9k$
$-2k \quad -2k$

$\frac{-6}{7} = \frac{7k}{7}$

$\boxed{k = \frac{-6}{7}}$

$64^{3k-2} = 16^{-3k}$

$(4^3)^{3k-2} = (4^2)^{-3k}$

$4^{9k-6} = 4^{-6k}$

$9k - 6 = -6k$
$+6k \quad +6k$

$15k - 6 = 0$
$+6 \quad +6$

$\frac{15k}{15} = \frac{6}{15}$

$k = \frac{6 \div 3}{15 \div 3} = \boxed{\frac{2}{5}}$

Graphing Exponents

$y = \frac{1}{3}\left(\frac{1}{2}\right)^x$

amplitude: $\frac{1}{3}$ — base: $\frac{1}{2}$

$y = ab^x$

$y = ab^{(x-h)} + k$

(h, k) movement

$y = ab^x$ — base: b

$b < 1$ $\quad 0 \longleftrightarrow 1$

decreasing

$b > 1$ $\quad 1 \longleftrightarrow \infty$

increasing

$y = \frac{1}{3}\left(\frac{1}{2}\right)^x$

Exponent = $\boxed{0}$

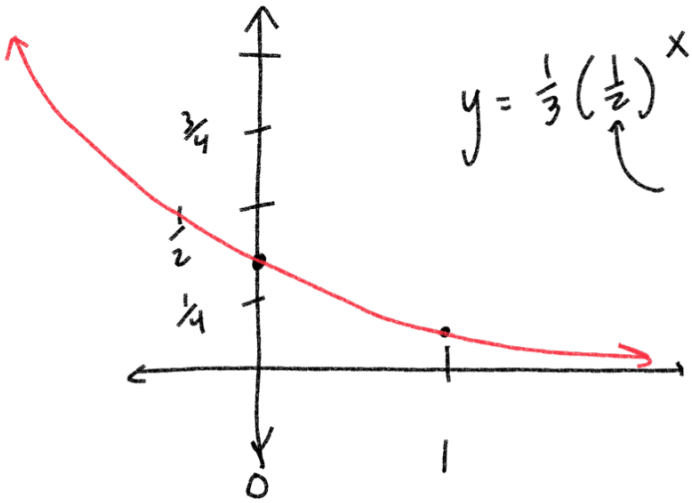
$\frac{1}{3}\left(\frac{1}{2}\right)^0 = \frac{1}{3}(1) = \frac{1}{3}$

$0, \frac{1}{3}$

Exponent = $\boxed{1}$

$\frac{1}{3}\left(\frac{1}{2}\right)^1 = \frac{1}{3}\left(\frac{1}{2}\right) = \frac{1}{6}$

$\left(1, \frac{1}{6}\right)$



$y = \frac{1}{3}\left(\frac{1}{2}\right)^x$ — base: $\frac{1}{2}$

$\frac{1}{2} < 1$

decreasing

1.) $y = 5 \cdot \left(\frac{1}{4}\right)^x$

increasing/decreasing

decreasing

$x = 0 \quad y = 5$

$x = 1 \quad y = \frac{5}{4}$

2.) $y = \frac{1}{3}(2)^x$

increasing

$x = 0 \quad y = \frac{1}{3}$

$x = 1 \quad y = \frac{2}{3}$

Compound Interest

non-continuous

$$A = A_0\left(1 + \frac{r}{n}\right)^{nt}$$

A = total amount

A_0 = initial amount

r = rate (decimal)

t = time

n = compound frequency

daily $n = 365$

monthly $n = 12$

quarterly $n = 4$

$A_0 = 2000$

$r = 8\%$

compounded semi-annually $n = 2$

$t = 6$ years

$$A = 2000\left(1 + \frac{0.08}{2}\right)^{2(6)}$$

Compound continuously $n = \infty$

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

$$\left(1 + \frac{1}{\infty}\right)^{\infty} = 2.7172\ldots = e$$

e is the natural number

Continuous Compounding Interest

$$\boxed{A = Pe^{rt}}$$

P = principal (initial amount)

P = $8,615

r = 2%

t = 11 years

$$A = 8{,}615\, e^{(0.02)(11)}$$

$$= 8{,}615\, e^{0.22}$$

$e^0 = 1$

$\boxed{\$10{,}734.95}$

P = $500,000

r = 6%

t = 26.82

Goal = $2,500,000

t = ?

$$P = e^{rt}$$

$$\frac{\$2{,}500{,}000}{500{,}000} = \frac{500{,}000\, e^{0.06t}}{500{,}000}$$

$$5 = e^{0.06t}$$

Logarithm is an exponent

$\log_{10} 10{,}000 = x$ ← exponent

↑ base

$10^x = 10{,}000$

$10^4 = 10{,}000$

$10^0 = 1$

$10^1 = 10$

$10^2 = 100$

$10^3 = 1000$

$10^4 = 10{,}000$

Logarithmic Form

$\log_2 64 = x$

Exponential form → $2^x = 64$ $\quad x = 6$

$2^6 = 64$

Logarithmic form

$\log_3 81 = x$

Exponential form

$3^x = 81$ $\quad x = 4$

$3^4 = 81$

Exponential form → Logarithmic form

$2^x = 32$ → $\log_2 32 = x$

$4^9 = x$ → $\log_4 x = 9$

$\log_5 \frac{1}{125} = x$ base is always the base

$5^x = \frac{1}{125}$ $\quad 5^x = 5^{-3}$

$5^x = \frac{1}{5^3}$ $\quad \boxed{x = -3}$

log form $\quad \log_a 19 = \frac{1}{2}$ $\quad$ exp form $\quad a^{\frac{1}{2}} = 19$

$(\sqrt{a})^2 = (19)^2$

$\boxed{a = 361}$

$(a^{\frac{1}{2}})^2 = (19)^2$

$a^{\frac{2}{2}} = 361$

$a^1 = 361$

$\log_9 y = 81$ $\quad 9^{81} = y$

$\log_2 \frac{1}{8}$

$2^x = \frac{1}{8}$

$2^x = 2^{-3}$

$\boxed{x = -3}$

$\log_3 27$

$3^x = 27$

$3^x = 3^3$

$\boxed{x = 3}$

$\log_6 \frac{1}{216}$

$6^x = \frac{1}{216}$

$6^x = 6^{-3}$

$\boxed{x = -3}$

$\log_x x^b = b$

$x^b = x^b$

$\log_a a^{12} = x$

$a^x = a^{12}$

$x = 12$

$\log_a a^{12} = 12$

$\log 8 = \log_{10} 8$

understood 10

$\log 8$

$10^x = 8$

$\ln \rightarrow$ natural log

$\ln = \log_e$ ← base e

$\log_8 27$

$8^x = 27$

Change of Base

$\log 8^x = \log 27$

$x \log 8 = \log 27$

$\frac{x \log 8}{\log 8} = \frac{\log 27}{\log 8}$

$$x = \frac{\log 27}{\log 8}$$

$$\log_2 76 = X$$

$$\log 2^X = \log 76$$

$$X \log 2 = \log 76$$

$$\frac{X \log 2}{\log 2} = \frac{\log 76}{\log 2}$$

$$X = \frac{\log 76}{\log 2}$$

6.25