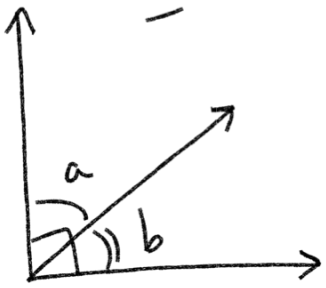
TTH-PT Pre-Calculus: Trigonometry Session 5

Complementary Angles



$\measuredangle a + \measuredangle b = 90°$

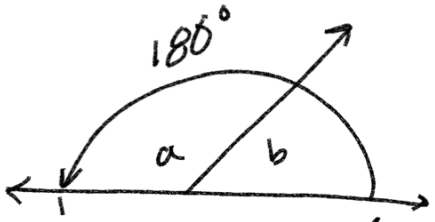
$\measuredangle a = 18°$

$18° + \measuredangle b = 90°$

$-18° \quad -18°$

$\boxed{\measuredangle b = 72°}$

Supplementary Angles



$\measuredangle a + \measuredangle b = 180°$

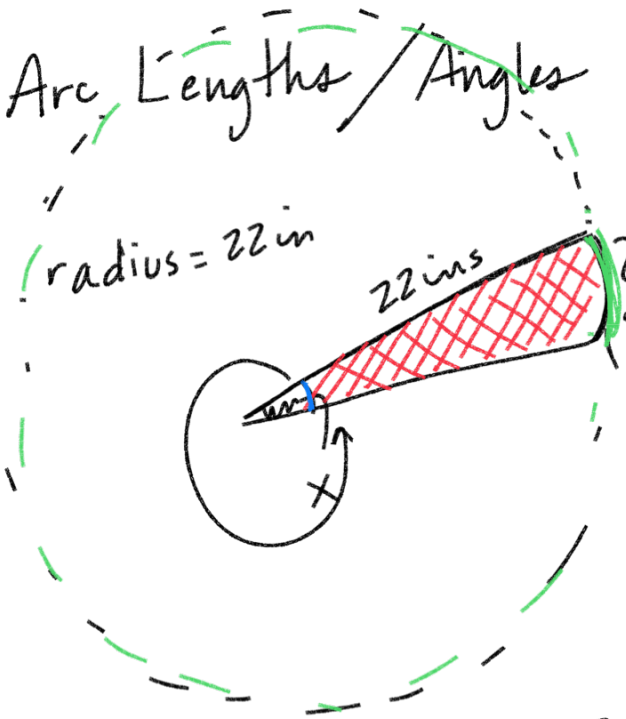
$\measuredangle a = 108°$

$108° + \measuredangle b = 180°$

$-108° \quad -108°$

$\boxed{\measuredangle b = 72°}$

Arc Lengths / Angles



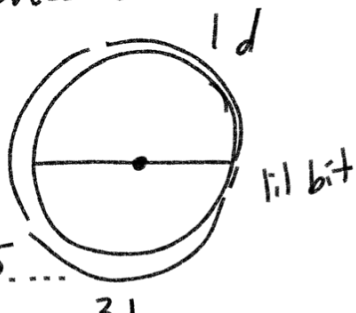
$\frac{\text{arc length}}{\text{circumference}} = \frac{X}{360°}$

$\frac{4in}{2\pi(22in)} = \frac{X}{360°}$

arc length Proportional

Circumference of Circle

$C = \pi d$



$\frac{C}{d} = \pi = 3.1415\ldots$

$\pi = 2 + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \cdots$

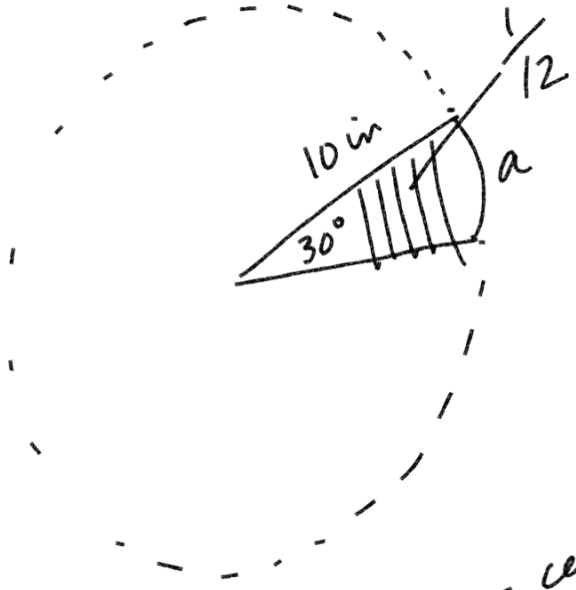
$\boxed{C = 2\pi r}$

$d = 2r$

$\frac{4in}{44\pi in} = \frac{X}{360°}$

$\frac{(360°)(4in)}{44\pi in} = \frac{(44\pi in)X}{44\pi in}$

$X = \frac{360°}{11\pi} \approx \boxed{10.4°}$



$r = 10$ in

arc length = s

$\theta = 30°$

$s = a$

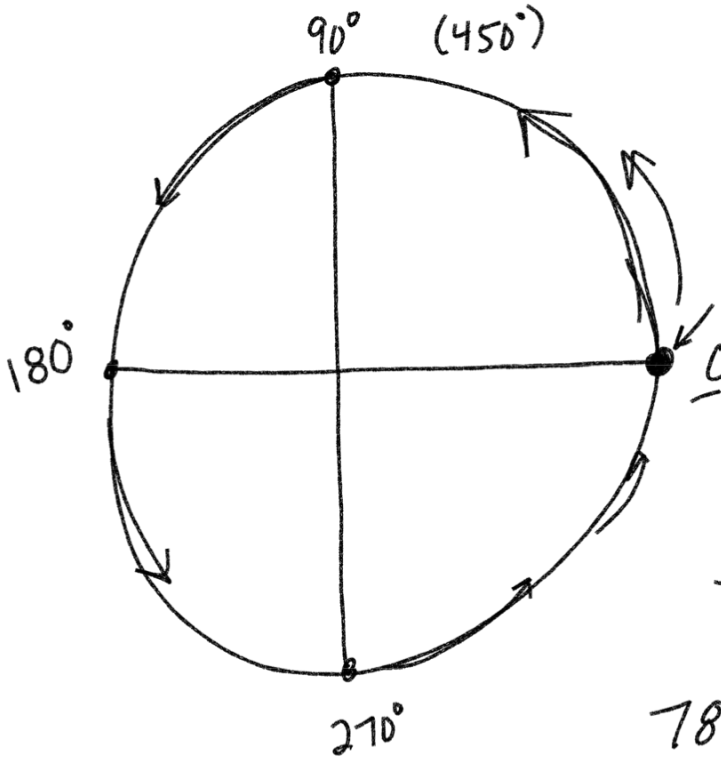
$$\frac{s}{2\pi r} = \frac{\theta}{360}$$

$$\frac{a}{2\pi(10\text{ in})} = \frac{30°}{360°}$$

$s = r\theta$ ← central angle (in radians)

$$\frac{600\pi\text{ in}}{360} = \frac{360a}{360}$$

$$a = \frac{600\pi}{360}\text{ in} = \boxed{5.2\text{ in}}$$



$450° = 90°$

reference angles

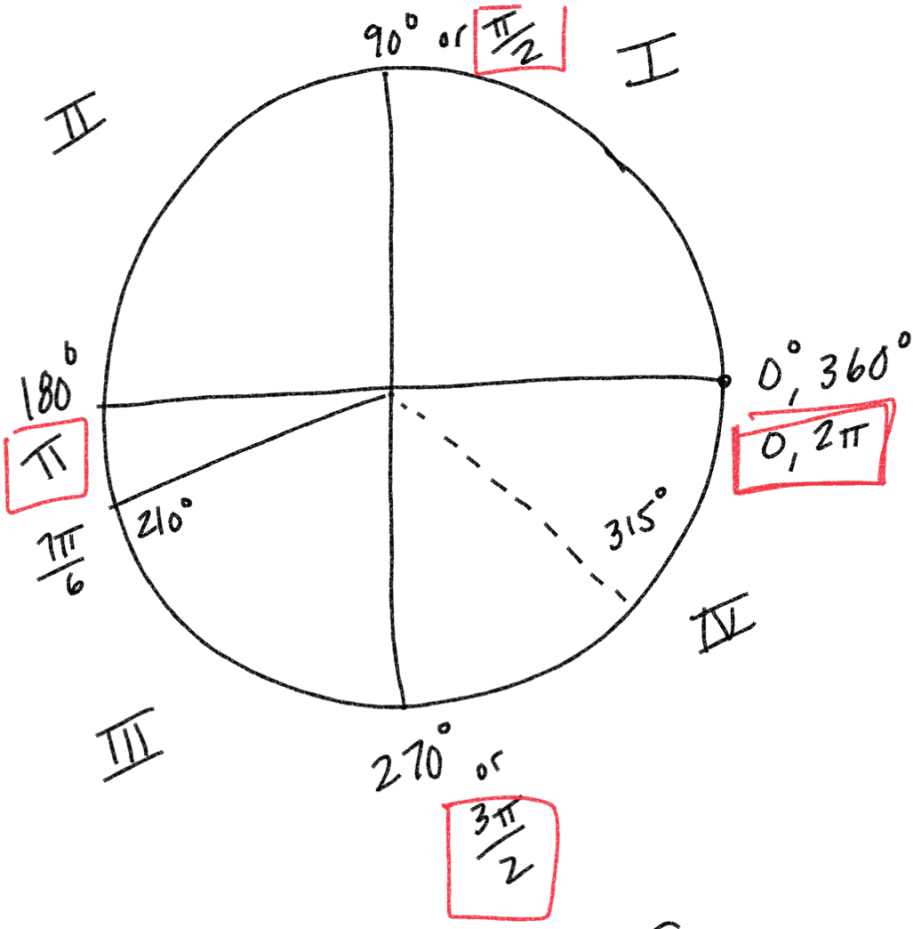
$720°$

$-360°$

$780°$ reference angle

$780° - 360° = 420° - 360° = \boxed{60°}$

$-320° + 360° = 360° - 320° = \boxed{40°}$



Unit Circle

radius = 1

Circumference of a unit circle

$C = 2\pi r \quad r = 1$

$C = 2\pi(1)$

$C = 2\pi$

Convert to radians $\left\{ \frac{180^\circ}{\pi} \text{ or } \frac{\pi}{180^\circ} \right\}$ because $180^\circ = \pi$

$30^\circ * \frac{\pi}{180^\circ} = \frac{30\pi}{180} \overset{\div 30}{=} \frac{\pi}{6}$

Convert to degrees

$\frac{\pi}{4} * \frac{180^\circ}{\pi} = \frac{180^\circ}{4} = 45^\circ$

$315^\circ \qquad 315^\circ * \frac{\pi}{180^\circ} = \frac{315\pi}{180} \overset{\div 5}{=} \frac{63\pi}{36} \overset{\div 9}{=} \frac{7\pi}{4}$

$\frac{7\pi}{6} \qquad \frac{7\pi}{6} * \frac{180^\circ}{\pi} = 210^\circ$

Arc Length (s)

$s = \theta r$ $\quad s = \left(\frac{\pi}{8}\right)(6) = \frac{6\pi}{8} \div 2 = \frac{3\pi}{4}$

$\theta = \frac{\pi}{8}$

$r = 6$

$\frac{s}{2\pi r} = \frac{\theta}{2\pi}$

$\frac{\text{arc length}}{\text{circumference}} = \frac{x}{360^\circ}$

$r\left(\frac{s}{r}\right) = (\theta)r$ $\quad s = \theta r$

Trigonometric Ratios

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin\theta = \frac{b}{c}$

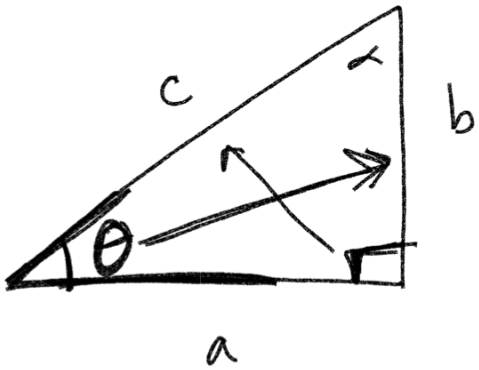
$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\cos\theta = \frac{a}{c}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{b}{a}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{opp}{hyp}}{\frac{adj}{hyp}}$ $\quad \frac{opp}{hyp} \div \frac{adj}{hyp}$ $\quad \frac{opp}{hyp} * \frac{hyp}{adj} = \frac{opp}{adj}$

Right Triangle



SOH $\quad \sin\theta = \frac{opp}{hyp}$

CAH $\quad \cos\theta = \frac{adj}{hyp}$

TOA $\quad \tan\theta = \frac{opp}{adj}$

Cosecant $= \frac{1}{\sin\theta}$ secant $= \frac{1}{\cos\theta}$

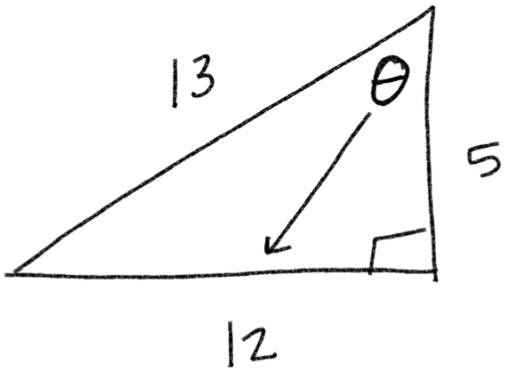
$\left[\csc\theta = \frac{1}{\sin\theta}\right]$ $\left[\sec\theta = \frac{1}{\cos\theta}\right]$

or

$\sin\theta = \frac{1}{\csc\theta}$ or $\cos\theta = \frac{1}{\sec\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ or $\left[\cot\theta = \frac{1}{\tan\theta}\right]$

$\sin\theta = \frac{12}{13}$ $\frac{opp}{hyp}$



$\cos\theta = \frac{adj}{hyp} = \frac{5}{13}$

$\tan\theta = \frac{opp}{adj} = \frac{12}{5}$

$\sec\theta = \frac{1}{\cos\theta}$ inverse of $\cos\theta = \frac{13}{5}$

$\csc\theta = \frac{1}{\sin\theta}$ inverse of $\sin\theta = \frac{13}{12}$

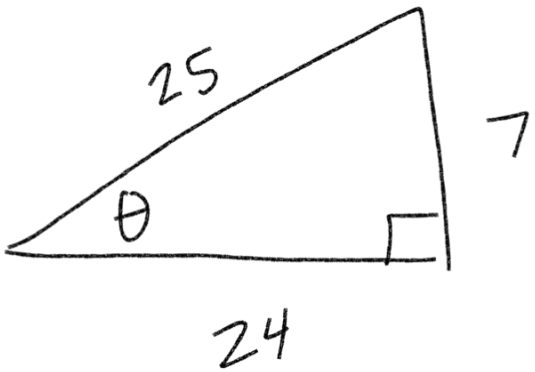
$\cot\theta = \frac{1}{\tan\theta}$ inverse of $\tan\theta = \frac{5}{12}$

$\cos\theta = \frac{24}{25}$

$\sin\theta = \frac{7}{25}$

$\tan\theta = \frac{7}{24}$

$\csc\theta = \frac{25}{7}$

$\sec\theta = \frac{25}{24}$

$\cot\theta = \frac{24}{7}$