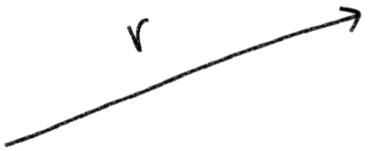
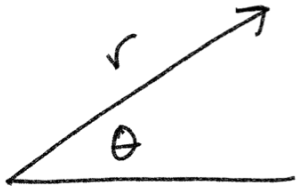
T TH- Pre-Calculus : Trigonometry Session 19

Vector → magnitude and a direction

resultant / radius



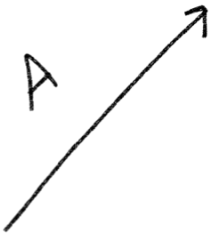
Polar Coordinate



(r, θ)

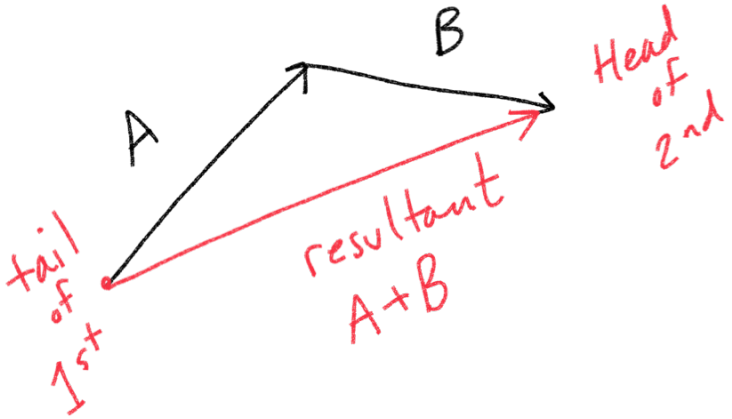
magnitude

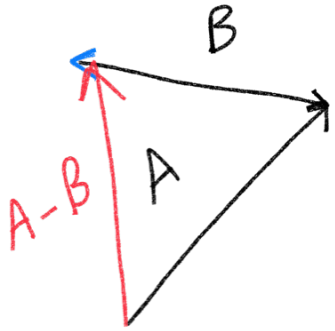
direction



$A + B$

$A - B$

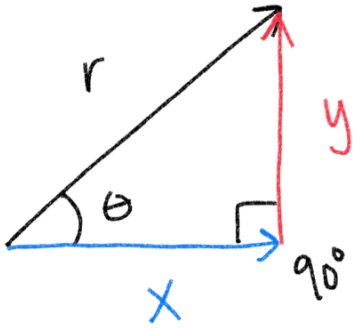




Vectors

x- component

y- component



Right Triangle

$\cos\theta = \frac{x}{r}$

$x = r\cos\theta$

$\sin\theta = \frac{y}{r}$

$y = r\sin\theta$

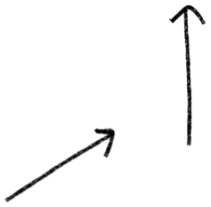
$r = 1$

$x^2 + y^2 = 1^2$

$x^2 + y^2 = 1$

$(r\cos\theta)^2 + (r\sin\theta)^2 = 1$

$$\cos^2\theta + \sin^2\theta = 1$$



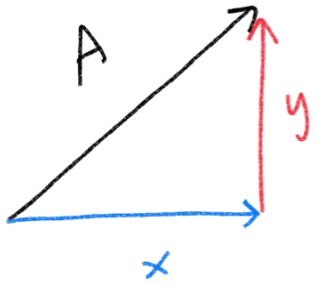
$y = r\sin\theta$

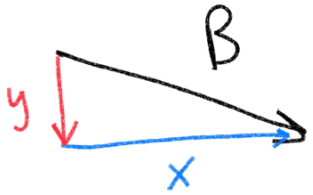
$y = 5000N \sin 70°$

$= 4698\ N$

or

1054.8 lbs





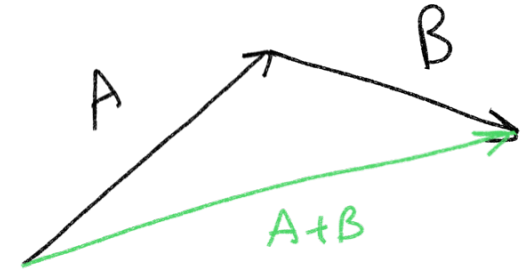
$A+B$

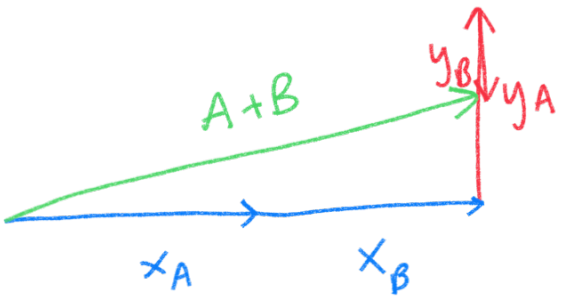
$A+B$

$\langle x_A, y_A \rangle + \langle x_B + y_B \rangle$

$\langle x_A + x_B, \; y_A + y_B \rangle$

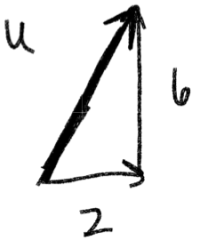
$\vec{u} + \vec{v}$ (vector)



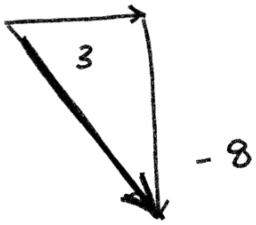


$u: \langle 2, 6 \rangle$

$v: \langle 3, -8 \rangle$



v



$\vec{u} + \vec{v}$ $\quad \langle 2+3, \; 6+(-8) \rangle$

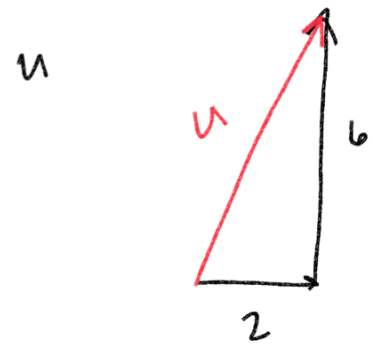
$\boxed{\langle 5, -2 \rangle}$

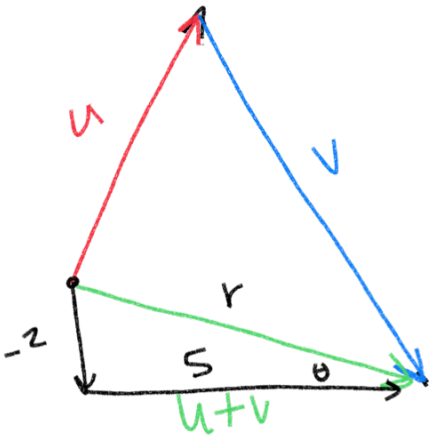
$u: \langle 2, 6 \rangle$

$v: \langle 3, -8 \rangle$

$\vec{u} + \vec{v}$ $\quad \langle 2+3, 6+(-8) \rangle$

$\boxed{\langle 5, -2 \rangle}$





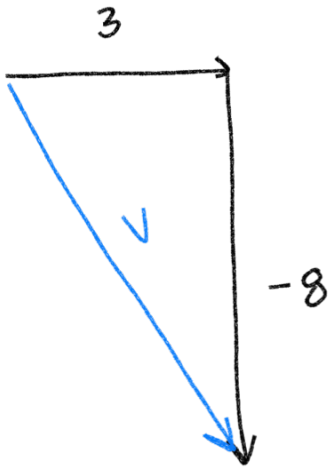
$\langle 5, -2 \rangle$

v

resultant (hypotenuse)

$r = \sqrt{5^2 + (-2)^2}$

$\sqrt{25+4} = \sqrt{29}$



$\tan\theta = \frac{-2}{5}$

$\theta = \tan^{-1}\left(\frac{-2}{5}\right) = -21.8°$

$$\begin{array}{r} 59 \\ 360.10 \\ -\ 21.8 \\ \hline 338.2° \end{array}$$

Vector $\vec{u} + \vec{v}$

$(\sqrt{29}, 338.2°)$

$\boxed{(5.38, 338.2°)}$

magnitude direction

$\vec{A}$: $\langle 3, 5 \rangle$ (x_A y_A)

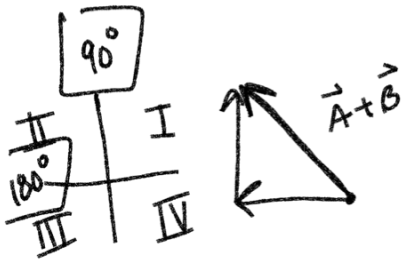
(magnitude, direction)

$\vec{B}$: $\langle -6, -1 \rangle$ (x_B y_B)

$\vec{A} + \vec{B}$

$\langle 3+(-6), 5+(-1) \rangle$

$\langle -3, 4 \rangle$



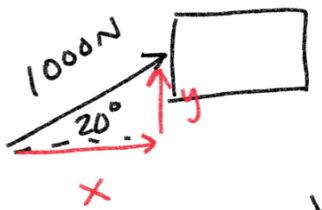
$r = \sqrt{(-3)^2 + (4^2)} = \sqrt{9+16} = \sqrt{25} = 5$

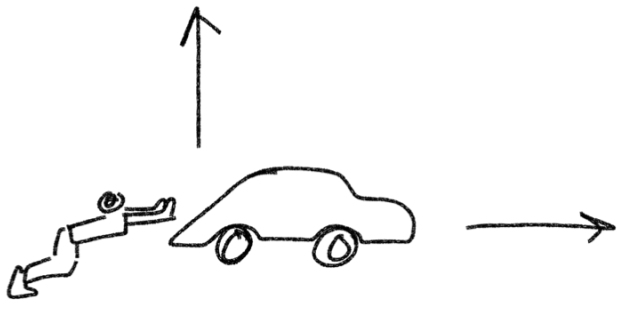
$\theta = \tan^{-1}\left(\frac{4}{-3}\right) = -53.13°$

$(5, 126.87)$

$180 - 53.13 = 126.87$

Dot Product





$X = r\cos\theta$

$= (1000\ N)\cos 20° = 939.7\ N$

$\vec{u} : \langle 2, 6 \rangle$

$\vec{v} : \langle 3, -8 \rangle$

Dot Product

$\vec{u} \cdot \vec{v}$

$$(\vec{u}_x)(\vec{v}_x) + (\vec{u}_y)(\vec{v}_y)$$

$$(2)(3) + (6)(-8)$$

$$6 + (-48)$$

$$\boxed{-42}$$

Angle Between Vectors

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| * |\vec{v}|}$$

$\vec{u} \cdot \vec{v} = -42$

$|\vec{u}|$ $\quad \vec{u} : \langle 2, 6 \rangle$

$$|\vec{u}| = \sqrt{2^2 + 6^2} = \sqrt{40}$$

$|\vec{v}|$ $\quad \vec{v} : \langle 3, -8 \rangle$

$$|\vec{v}| = \sqrt{3^2 + (-8)^2} = \sqrt{73}$$

$$\cos\theta = \frac{-42}{\sqrt{40} \cdot \sqrt{73}}$$

$$\theta = \cos^{-1}\left(\frac{-42}{\sqrt{40} \cdot \sqrt{73}}\right)$$

$$\theta = 142.5^\circ$$