Calculus & Trigonometry

Chapter 6 Pre-Test

1.) (20 pts total, 5 pts each) Simplify each of the following trigonometric expressions.

a) $(sin^2 x)(cot^2 x + 1)$

b) $(sin\, x - cos\, x)(sin\, x + cos\, x)$

c) $\frac{1 - cos^4 x}{1 + cos^2 x}$

d) $1 - \frac{sin^2 x}{1 - cos\, x}$

2.) (40 pts total, 5 pts each) Verify each of the following trigonometric identities.

a) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

b) $\tan x + \cot x = \csc x \sec x$

c) $\frac{2 - \sin^2 x}{\cos x} = \sec x + \cos x$

d) $\frac{1}{\csc^2 x} + \frac{1}{\sec^2 x} = 1$

$$\frac{1}{\frac{1}{\sin^2 x}} = 1 * \frac{\sin^2 x}{1} = \sin^2 x$$

$$\frac{1}{\frac{1}{\sin^2 x}} + \frac{1}{\frac{1}{\cos^2 x}} = 1$$

yes!

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

1.) Where is $\sin^2 x + \cos^2 x = 1$?

2.) Common denominator/ Factoring

3.) Difference of squares

e) $\frac{1}{1 - \sin x} + \frac{1}{1 + \sin x} = 2\sec^2 x$

$$\frac{(1+\sin x)}{(1+\sin x)}\frac{1}{1-\sin x} + \frac{1}{1+\sin x}\frac{(1-\sin x)}{(1-\sin x)} = \frac{2}{\cos^2 x}$$

$$\frac{1+\sin x}{1-\sin^2 x} + \frac{1-\sin x}{1-\sin^2 x}$$

FOIL

$$(1+\sin x)(1-\sin x)$$

$$1 - \cancel{\sin x} + \cancel{\sin x} - \sin^2 x$$

$$1 - \sin^2 x$$

$$\frac{1+\sin x + 1 - \sin x}{1 - \sin^2 x} = \frac{2}{\cos^2 x} = 2\left(\frac{1}{\cos^2 x}\right)$$

$$\boxed{2\sec^2 x}$$

f) $\frac{\sin^2 x}{1 - \cos x} = 1 + \cos x$ ✓

"complex conjugate"

$$\frac{(\sin^2 x)(1+\cos x)}{(1-\cos x)(1+\cos x)}$$

$$\frac{\sin^2 x + \sin^2 x \cos x}{1 - \cos^2 x} = \frac{\sin^2 x + \sin^2 \cos x}{\sin^2 x}$$

$$\frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}} + \frac{\cancel{\sin^2 x}\cos x}{\cancel{\sin^2 x}}$$

$$\boxed{1 + \cos x}$$

$$\frac{\sin^2 x}{1 - \cos x} = \frac{1 - \cos^2 x}{1 - \cos x}$$

$$\frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{1-\cos x}}$$

complex conjugate

g) $\sec x + \tan x = \frac{1}{\sec x - \tan x}$

{split num nums, don't split den dens}

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x} = \frac{1}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}$$

Keep, Change, Flip!

$$\frac{(\cos x)(1+\sin x)}{(1-\sin x)(1+\sin x)} = \frac{1}{\frac{1-\sin x}{\cos x}}$$

$$1 \div \frac{1-\sin x}{\cos x}$$

$$1 * \frac{\cos x}{1-\sin x}$$

$$\frac{\cos x + \sin x \cos x}{1-\sin^2 x} =$$

$$\frac{\cancel{\cos x} + \sin x \cancel{\cos x}}{\cos^{\cancel{2}} x}$$

$$\frac{1+\sin x}{\cos x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$\boxed{\sec x + \tan x}$$

h) $\frac{\cos^2 x + 1 + \sin x}{\cos^2 x + 3} = \frac{1 + \sin x}{2 + \sin x}$

clues to factor

"Hint"

$\cos^2 x = 1 - \sin^2 x$

"Hint #2"

factor a poly gumgum

3.) (10 pts total, 5 pts each) Write each expression as a single trigonometric function.

a) $\sin(4x)\sin(3x) + \cos(4x)\cos(3x)$

b) $sin\ x\ cos\ (8x)\ -\ cos\ x\ sin\ (8x)$

4.) (10 pts total, 5 pts each) Use double angle identities to solve each of the following.

a) $If\ cos\ x\ =\ \frac{5}{24}\ and\ sin\ x\ <\ 0,\ find\ tan\ (2x)$

b) $sin\ 15°\ cos\ 15°$

Half or Double

5.) (10 pts total, 2 pts each) Find the exact values of each problem

a) $\cos 15°$

$A = 30$

$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$

$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$

$\tan\left(\frac{A}{2}\right) = \frac{\pm\sqrt{1-\cos A}}{\sqrt{1+\cos A}}$

$\cos 15° = \cos\frac{30}{2}$

$2(15) = \left(\frac{A}{2}\right)2$

$30 = A$

$\cos 15° = \pm\sqrt{\frac{1+\cos A}{2}}$

$\pm\sqrt{\frac{1+\cos 30}{2}}$

$\pm\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{1}{2}+\frac{\sqrt{3}}{4}}$

$\pm\sqrt{\frac{2}{4}+\frac{\sqrt{3}}{4}}$

$\pm\sqrt{\frac{2+\sqrt{3}}{4}}$

$\boxed{\pm\frac{\sqrt{2+\sqrt{3}}}{2}}$

b) $\tan 202.5°$

$\tan\frac{405}{2}$

$\tan A = \tan 405$

reference angle $45° =$

$\tan 202.5 = \pm\frac{\sqrt{1-\cos 45}}{\sqrt{1+\cos 45}} = \frac{\sqrt{1-\frac{\sqrt{2}}{2}}}{\sqrt{1+\frac{\sqrt{2}}{2}}} \quad \frac{\sqrt{\frac{2}{2}-\frac{\sqrt{2}}{2}}}{\sqrt{\frac{2}{2}+\frac{\sqrt{2}}{2}}}$

$\geq 90°$

$\frac{\sqrt{\frac{2-\sqrt{2}}{2}}}{\sqrt{\frac{2+\sqrt{2}}{2}}}$

$\boxed{\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}}$

6.) (5 pts total) Write the product as a sum or difference.

$4\sin(3x)\sin(4x)$

$\sin A \sin B = \frac{[\cos(A-B) - \cos(A+B)]}{2}$

$\frac{\cos(3x-4x) - \cos(3x+4x)}{2}$

$\frac{\cancel{4}^{2}}{\cancel{2}}\left(\cos(-x) - \cos 7x\right)$

$2\cos(-x) - \cos(7x)$

(even)

$\cos x$

7.) (5 pts total) Write the expression as a product.

$$\cos(8x) + \cos(3x)$$

$8x \quad 3x$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\boxed{2\cos\left(\frac{11x}{2}\right)\cos\left(\frac{5x}{2}\right)}$$