

Calculus & Trigonometry

Chapter 6 Pre-Test

1.) (20 pts total, 5 pts each) Simplify each of the following trigonometric expressions.

a) $(\sin^2 x)(\cot^2 x + 1)$

$$\cancel{\sin^2 x} \left(\frac{\cos^2 x}{\cancel{\sin^2 x}} + 1 \right)$$

$$\cos^2 x + \sin^2 x = \boxed{1}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

b) $(\sin x - \cos x)(\sin x + \cos x)$

FOIL

$$\sin^2 x + \cancel{\sin x \cos x} - \cancel{\sin x \cos x} - \cos^2 x$$

$$\boxed{\sin^2 x - \cos^2 x}$$

$$1 - \cos^4 x = (1 + \cos^2 x)(1 - \cos^2 x)$$

$$\frac{(1 + \cancel{\cos^2 x})(1 - \cos^2 x)}{1 + \cancel{\cos^2 x}}$$

$$1 - \cos^2 x =$$

$$\boxed{\sin^2 x}$$

difference of squares
hint

c) $\frac{1 - \cos^4 x}{1 + \cos^2 x}$

$$\begin{array}{r} \sin^2 x + \cos^2 x = 1 \\ - \cos^2 x \quad - \cos^2 x \end{array}$$

$$\sin^2 x = 1 - \cos^2 x$$

d) $1 - \frac{\sin^2 x}{1 - \cos x}$

$$1 - \frac{1 - \cos^2 x}{1 - \cos x}$$

$$= 1 - \frac{(1 + \cos x)(1 - \cos x)}{1 - \cos x}$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\frac{1 - \cos x}{1 - \cos x} - \frac{\sin^2 x}{1 - \cos x}$$

$$1 - 1 - \cos x = \boxed{-\cos x}$$

$$\frac{1 - \cos X - \sin^2 X}{1 - \cos X}$$

$$\frac{1 - \cos X - (1 - \cos^2 X)}{1 - \cos X}$$

$$\frac{-\cos X + \cos^2 X}{1 - \cos X}$$

2.) (40 pts total, 5 pts each) Verify each of the following trigonometric identities.

a) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

$$\frac{-\cos X (1 - \cos X)}{1 - \cos X}$$

$$(\sin x + \cos x)(\sin x + \cos x) + (\sin x - \cos x)(\sin x - \cos x)$$

$$\sin^2 x + \cancel{\sin x \cos x} + \cancel{\sin x \cos x} + \cos^2 x$$

$$\sin^2 x - \cancel{\sin x \cos x} - \cancel{\sin x \cos x} + \cos^2 x$$

$$2 \sin^2 x + 2 \cos^2 x = 2(\sin^2 x + \cos^2 x)$$

$\sin^2 x$
 $\sin 2x \rightarrow$ double angle

b) $\tan x + \cot x = \csc x \sec x$

$$2(1) = \boxed{2}$$

$$\frac{\sin x (\sin x)}{\sin x (\cos x)} + \frac{\cos x (\cos x)}{\sin x (\cos x)}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = \left(\frac{1}{\sin x}\right) \left(\frac{1}{\cos x}\right)$$

$$\boxed{\csc x \sec x}$$

c) $\frac{2 - \sin^2 x}{\cos x} = \sec x + \cos x$

$$\sin^2 x + \cos^2 x = 1$$

$$-\cos^2 x - \cos^2 x$$

nope!

$$\sin^2 x = 1 - \cos^2 x$$

$$\frac{2 - (1 - \cos^2 x)}{\cos x}$$

$$\frac{2 - 1 + \cos^2 x}{\cos x}$$

$$\frac{1 + \cos^2 x}{\cos x} = \frac{1}{\cos x} + \frac{\cos^2 x}{\cos x}$$

$$\sec x + \cos x$$

$$d) \frac{1}{\csc^2 x} + \frac{1}{\sec^2 x} = 1$$

$$e) \frac{1}{1 - \sin x} + \frac{1}{1 + \sin x} = 2\sec^2 x$$

$$f) \frac{\sin^2 x}{1 - \cos x} = 1 + \cos x$$

$$g) \sec x + \tan x = \frac{1}{\sec x - \tan x}$$

$$h) \frac{\cos^2 x + 1 + \sin x}{\cos^2 x + 3} = \frac{1 + \sin x}{2 + \sin x}$$

3.) (10 pts total, 5 pts each) Write each expression as a single trigonometric function.

$$a) \overset{\theta}{\sin(4x)} \overset{\phi}{\sin(3x)} \overset{\downarrow}{+} \overset{\uparrow}{\cos(4x)} \cos(3x)$$

$$\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$$

$$\cos(\theta - \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi$$

$$\cos(4x - 3x) = \boxed{\cos x}$$

b) $\sin x \cos(8x) - \cos x \sin(8x)$

$\sin(x - 8x)$

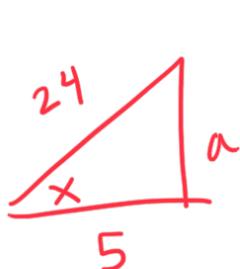
$\sin(-7x)$

$\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$

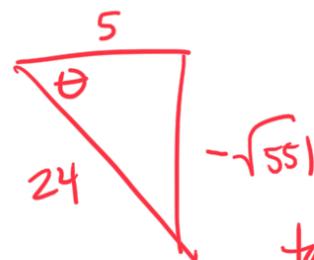
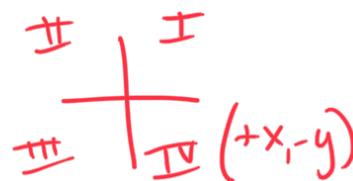
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4.) (10 pts total, 5 pts each) Use double angle identities to solve each of the following.

a) If $\cos x = \frac{5}{24}$ and $\sin x < 0$, find $\tan(2x)$



$5^2 + a^2 = 24^2$
 $25 + a^2 = 576$
 $-25 \quad -25$
 $\sqrt{a^2} = \sqrt{551}$
 $a = \pm \sqrt{551}$



$\tan \theta = \frac{5}{-\sqrt{551}}$

b) $\sin 15^\circ \cos 15^\circ$

product to sum

$\tan 2\theta$ Double Angle

$\frac{\sin(\theta + \phi) + \sin(\theta - \phi)}{2}$

$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

$\frac{\sin(15+15) + \sin(15-15)}{2}$

$= \frac{2 \left(-\frac{5}{\sqrt{551}}\right)}{1 - \left(-\frac{5}{\sqrt{551}}\right)^2}$ (4.5)

$\frac{\sin 30 + \sin 0}{2}$

$\frac{\frac{1}{2} + 0}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$

5.) (10 pts total, 2 pts each) Find the exact values of each problem

a) $\cos 15^\circ$

b) $\tan 202.5^\circ$

6.) (5 pts total) Write the product as a sum or difference.

$$4\sin(3x)\sin(4x)$$

7.) (5 pts total) Write the expression as a product.

$$\cos(8x) + \cos(3x)$$