TTH-PT Pre-Calculus: Trigonometry Session 15

Double Angle Identity

$$\sin(2A) = 2\sin A\cos A$$

$$\cos(2A) = \begin{cases} \cos^2 A - \sin^2 A \leftarrow \\ 1 - 2\sin^2 A \leftarrow \\ 2\cos^2 A - 1 \leftarrow \end{cases}$$

$$\left\{\tan(2A) = \frac{2\tan A}{1-\tan^2 A}\right\}$$

$$\left\{\sin(2x) = 2\sin x\cos x\right\}$$

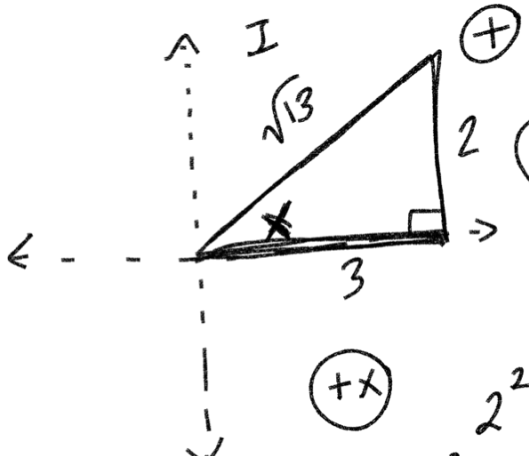
$$\sin 6x = 2\sin 3x\cos 3x$$

$$\cos(2A) = (\cos A)^2 - (\sin A)^2$$

If $\left[\sin x = \frac{2}{\sqrt{13}}\right]$ and $\left[\cos x > 0\right]$

Find $\cos 2x$

$$\sin x = \frac{\text{opp}}{\text{hyp}} \rightarrow \frac{2}{\sqrt{13}}$$



pythagorean theorem

$$2^2 + x^2 = (\sqrt{13})^2$$

$$4 + x^2 = 13$$

$$-4 \quad -4$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = \pm 3 = 3$$

$$\cos(2x) = (\cos x)^2 - (\sin x)^2$$

$$= \left(\frac{3}{\sqrt{13}}\right)^2 - \left(\frac{2}{\sqrt{13}}\right)^2$$

$$\frac{9}{13} - \frac{4}{13} = \boxed{\frac{5}{13}}$$

$$\cos(2x) = 1 - 2(\sin x)^2$$

$$= 1 - 2\left(\frac{2}{\sqrt{13}}\right)^2$$

$$1 - 2\left(\frac{4}{13}\right)$$

$$1 - \frac{8}{13}$$

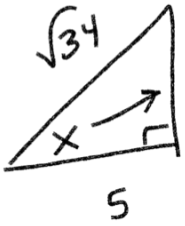
$$\frac{13}{13} - \frac{8}{13} = \boxed{\frac{5}{13}}$$

$\cos x = \frac{5}{\sqrt{34}}$ $\quad \sin x > 0$ $\quad$ Find $\sin(2x)$

$(+x, +y)$

$\sin 2x = \{2 \sin x \cos x\}$

Not drawn to scale



$y = 3$

$5^2 + y^2 = (\sqrt{34})^2$

$25 + y^2 = 34$

$-25 \qquad -25$

$\sqrt{y^2} = \sqrt{9}$

$y = \pm 3 \ldots (+3)$

$\sin x = \frac{\text{opp}}{\text{hyp}} = \frac{3}{\sqrt{34}}$

$\sin 2x = \boxed{2 \sin \frac{3}{\sqrt{34}} \cos \frac{5}{\sqrt{34}}}$

$2 \sin \frac{3\sqrt{34}}{34} \cos \frac{5\sqrt{34}}{34}$

$2 \sin 3x \cos 3x = \boxed{\sin 6x}$

Evaluate.

$\frac{2 \tan 15^\circ}{1 - \tan^2 15^\circ}$

$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

$= \tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

$2\cos^2\frac{\pi}{8} - 1 = \boxed{\frac{\sqrt{2}}{2}}$ " $2\cos^2\boxed{x} - 1 = \cos 2x$ (*2)

$= \cos 2\left(\frac{\pi}{8}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\sin(15^\circ)\cos(15^\circ) = \sin 2(15)$

$\sin 30^\circ = \boxed{\frac{1}{2}}$

Half Angles

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

Proof

$\sin\left(\frac{60}{2}\right) = \sin 30^\circ$

$\sin 30^\circ = \sin\frac{60}{2} = \pm\sqrt{\frac{1-\cos 60}{2}}$ (x2)

$\pm\sqrt{\frac{1-\frac{1}{2}}{2}} = \pm\sqrt{\frac{\frac{1}{2}}{2}}$

$\frac{1}{2} \div 2 = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$

$\pm\sqrt{\frac{1}{4}} = \pm\frac{\sqrt{1}}{\sqrt{4}} = \boxed{\pm\frac{1}{2}}$

If $\cos x = \frac{5}{13}$ and $\sin x < 0$, Find $\sin\left(\frac{x}{2}\right)$

Reference: $\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\frac{5}{13}}{2}} = \pm\sqrt{\frac{\frac{13}{13}-\frac{5}{13}}{2}} = \pm\sqrt{\frac{\frac{8}{13}}{2}}$$

$$\pm\sqrt{\frac{8}{26}} = \pm\sqrt{\frac{4}{13}} = \pm\frac{\sqrt{4}}{\sqrt{13}} = \pm\frac{2}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}}$$

$$\pm\frac{2\sqrt{13}}{13} \qquad \boxed{-\frac{2\sqrt{13}}{13}}$$

Product-to-Sum Identities

(Mult) $\longrightarrow$ (+)

$$\cos A * \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$4\cos(3x)\cos(2x)$

$A = 3x \qquad B = 2x$

$$\cos(3x)\cos(2x) = \frac{1}{2}\left[\cos(3x+2x) + \cos(3x-2x)\right]$$

$$\cos(3x)\cos(2x) = \frac{1}{2}\left[\cos(5x) + \cos x\right]$$

$$4\cos(3x)\cos(2x) = 4\left(\frac{1}{2}\left[\cos(5x) + \cos x\right]\right) = \boxed{2\cos(5x) + \cos x}$$

Proof

$\cos 60^\circ \cos 30^\circ$ $\quad A = 60^\circ \quad B = 30^\circ$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\sqrt{3}}{4}}$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\frac{1}{2}\left[\cos(60+30) + \cos[60-30)\right]$$

$$\frac{1}{2}\left[\cos 90^\circ + \cos 30^\circ\right]$$

$$\frac{1}{2}\left[0 + \frac{\sqrt{3}}{2}\right]$$

$$\frac{1}{2}\left[\frac{\sqrt{3}}{2}\right] = \boxed{\frac{\sqrt{3}}{4}}$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

1.) $-3\left[\sin(2x)\sin(4x)\right]$

$-3\left[\frac{1}{2}\left[\cos(2x-4x) - \cos(2x+4x)\right]\right.$

$\boxed{-\frac{3}{2}\left[\cos(-2x) - \cos(6x)\right]}$

or $-\frac{3}{2}\left[\cos(2x) - \cos(6x)\right]$

2.) $\left[\cos(10x)\sin(5x)\right]$

$\sin(10x+5x) + \sin(10x-5x)$

$\boxed{\sin(15x) + \sin(5x)}$

Even / Odd Functions

$\sin(-30^\circ) = -\sin 30^\circ$

odd → sine

$\cos(-30^\circ) = +\cos 30^\circ$

even → cos

$\cos(-2x) = \cos(2x)$

Sum-to-Product

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

Proof

$$\sin 30^\circ + \sin 60^\circ = 2\sin\left(\frac{30+60}{2}\right)\cos\left(\frac{30-60}{2}\right)$$

$$\frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$\frac{1+\sqrt{3}}{2}$$

$$2\sin\left(\frac{90}{2}\right)\cos\left(-\frac{30}{2}\right)$$

$$2\sin(45)\cos(-15)$$

$$2\left(\frac{\sqrt{2}}{2}\right)\cos(-15)$$

$$\sqrt{2}\cos(-15)$$

Half Angle

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

$\cos(-15)$

$\cos(-15) = \cos\left(\frac{-30}{2}\right)$

$\cos(45-60)$

$$\pm\sqrt{\frac{1+\cos(-30)}{2}} = \pm\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$$

$$\pm\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} \qquad \frac{2}{2}+\frac{\sqrt{3}}{2}=\frac{2+\sqrt{3}}{2}$$

$$\pm\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}=\pm\sqrt{\frac{2+\sqrt{3}}{4}}$$

$$\pm\frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\sqrt{2}\left(\frac{\sqrt{2+\sqrt{3}}}{2}\right)$$