TTH-PT Pre-Calculus: Trigonometry Session 14

1.) $\dfrac{\cos X + \sec X}{\sec X} = 1 + \cos^2 X$

$$\left\{ \frac{\cos X + \frac{1}{\cos X}}{\frac{1}{\cos X}} \right\} = 1 + \cos^2 X$$

$$\left(\cos X + \frac{1}{\cos X}\right) \div \frac{1}{\cos X}$$

$$\left(\cos X + \frac{1}{\cos X}\right) * \cos X$$

$$\cos^2 X + \frac{\cos X}{\cos X} = \cos^2 X + 1$$

2) $(\sec^2 x)(\sin^2 x) = \sec^2 x - 1$

$\sec^2 x \left(\quad \right)$

$\sin^2 x + \cos^2 x = 1$

$-\cos^2 x \quad -\cos^2 x$

$\sin^2 x = 1 - \cos^2 x$

$\sec^2 x (1 - \cos^2 x)$

$\sec^2 x - (\sec^2 x)(\cos^2 x)$

$\sec^2 x - \left(\frac{1}{\cos^2 x}\right)(\cos^2 x)$

$\sec^2 x - 1$

$(\sec^2 x)(\sin^2 x) = \sec^2 x - 1$

$\left(\frac{1}{\cos^2 x}\right)(\sin^2 x) = \sec^2 x - 1$

$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$

$\sin^2 x = 1 - \cos^2 x$

$+\cos^2 x \qquad +\cos^2 x$

$\sin^2 x + \cos^2 x = 1$

Sum & Difference of Cosine

Sum: $\cos(A+B) = \cos A \cos B - \sin A \sin B$

Difference: $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(15^\circ) = \cos(45-30)$ $A=45$ $B=30$

$(\cos 45)(\cos 30) + (\sin 45)(\sin 30)$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{2}}$

$\cos\left(\frac{7\pi}{12}\right)$

$\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$\cos\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right)$

Sum: $\left(\cos \frac{\pi}{3}^{60^\circ}\right)\left(\cos \frac{\pi}{4}^{45^\circ}\right) - \left(\sin \frac{\pi}{3}^{60^\circ}\right)\left(\sin \frac{\pi}{4}^{45^\circ}\right)$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}$

$\boxed{\frac{\sqrt{2}-\sqrt{6}}{2}}$

$(-,+)$

sum & difference of sine

sum: $\sin(A+B) = \sin A \cos B + \cos A \sin B$

difference: $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$$

$$\left(\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{4}\right) + \left(\cos\frac{\pi}{6}\right)\left(\sin\frac{\pi}{4}\right)$$

$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{2}}$$

Sum and Difference for Tangent

sum: $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

Difference: $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\tan 195^\circ = \tan(150+45)$ $\dfrac{\tan 150 + \tan 45}{1 - \tan 150 \tan 45}$

$$\frac{\tan 150 + \tan 45}{1 - \tan 150 \tan 45} = \frac{\tan 150 + 1}{1 - (\tan 150)(1)}$$

$$\tan 45^\circ = \frac{\sin 45^\circ}{\cos 45^\circ} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1 \qquad \frac{\tan 150^\circ + 1}{1 - \tan 150^\circ}$$

$$\tan 150^\circ = \frac{\sin 150^\circ}{\cos 150^\circ} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\frac{-\frac{\sqrt{3}}{3} + 1}{1 + \frac{\sqrt{3}}{3}}$$

Double Angles

$\sin(2\theta) = 2 \sin\theta \cos\theta$ LOOK UP

$\sin 60^\circ = \sin 2(30^\circ)$ $\quad \theta = 30^\circ$ $\quad \sin 60^\circ = \frac{\sqrt{3}}{2}$

$$\sin 2(30^\circ) = \underbrace{2(\sin 30^\circ)(\cos 30^\circ)}$$

$$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\sqrt{3}}{2}}$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

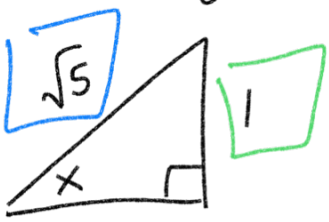
} look up

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

If $\sin x = \frac{1}{\sqrt{5}}$ and $\cos x < 0$ (negative) find $\sin 2x$

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$

$\sin x = \frac{1}{\sqrt{5}}$ → 1 opp side, $\sqrt{5}$ → hyp side

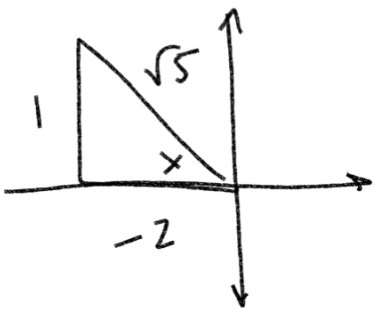


a

actually

Pythagorean Theorem

$a^2 + 1^2 = (\sqrt{5})^2$



$a^2 + 1 = 5$

$-1 \quad -1$

$\sqrt{a^2} = \sqrt{4}$

$a = \pm 2$

$\cos x =$

$\cos x = \frac{\text{adj}}{\text{hyp}} = \frac{-2}{\sqrt{5}}$

$\sin 2x = 2\sin x \cos x$

$2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{-2}{\sqrt{5}}\right) = 2\left(\frac{-2}{5}\right)$

$= -4/5$