TTH-PT Pre-Calculus : Trigonometry Session 13

$$(\cos x)(\sec x)(\cot^2 x) = \csc^2 x - 1$$

$$\left[(\cos x)\left(\frac{1}{\cos x}\right)\right]\left(\frac{\cos^2 x}{\sin^2 x}\right) = \frac{1}{\sin^2 x} - 1$$

$$1 = \frac{\sin^2 x}{\sin^2 x}$$

$$\frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}$$

$$\cos^2 x = 1 - \sin^2 x$$
$$+\sin^2 x \qquad +\sin^2 x$$
$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x}$$

$$\frac{\sin x + 1}{\sin x} = -\frac{\cot^2 x}{1 - \csc x}$$

$$\frac{\sin x + 1}{\sin x} = \left[\frac{-\dfrac{\cos^2 x}{\sin^2 x}}{1 - \dfrac{1}{\sin x}}\right]$$

$$\frac{-\cos^2 x}{\sin^2 x\left(1 - \frac{1}{\sin x}\right)}$$

$$\frac{-\cos^2 x}{\sin^2 x - \sin x}$$

$$\frac{-\cos^2 x}{\sin^2 x}\Big/\left(1 - \frac{1}{\sin x}\right)$$

$$\frac{20}{5}\Big/2 = \frac{4}{2} = 2$$

$$\frac{20}{5(2)} = \frac{20}{10} = 2$$

$$\sin^2 x + \cos^2 x = 1$$
$$-1 \quad -\cos^2 x \qquad -\cos^2 x$$
$$-1$$
$$-\cos^2 x = \sin^2 x - 1$$

$$\frac{-\cos^2 x}{\sin^2 x - \sin x} = \frac{\sin^2 x - 1}{\sin^2 x - \sin x} = \frac{(\sin x - 1)(\sin x + 1)}{\sin x(\sin x - 1)}$$

Difference of Squares

$$\frac{\sin x + 1}{\sin x}$$ ✓ true

$$\frac{\tan x - \cot x}{\tan x + \cot x} = \sin^2 x - \cos^2 x$$

$$\frac{\frac{\sin x}{\cos x}\left(\frac{\sin x}{\sin x}\right) - \frac{\cos x}{\sin x}\left(\frac{\cos x}{\cos x}\right)}{\frac{\sin x}{\cos x}\left(\frac{\sin x}{\sin x}\right) + \frac{\cos x}{\sin x}\left(\frac{\cos x}{\cos x}\right)} = \frac{\frac{\sin^2 x}{(\cos x)(\sin x)} - \frac{\cos^2 x}{(\cos x)(\sin x)}}{\frac{\sin^2 x}{(\cos x)(\sin x)} + \frac{\cos^2 x}{(\cos x)(\sin x)}}$$

$$\frac{\frac{\sin^2 x - \cos^2 x}{(\cos x)(\sin x)}}{\frac{\sin^2 x + \cos^2 x}{(\cos x)(\sin x)}}$$

$$\frac{\sin^2 x - \cos^2 x}{(\cos x)(\sin x)} * \frac{(\cos x)(\sin x)}{\sin^2 x + \cos^2 x} = \frac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x} = \frac{\sin^2 x - \cos^2 x}{1}$$

$$\sin^2 x - \cos^2 x$$

$$\frac{\sec x + \tan x}{\csc x + 1} = \tan x$$

$$\frac{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}{\frac{1}{\sin x} + \frac{\sin x}{\sin x}} = \frac{\frac{1 + \sin x}{\cos x}}{\frac{1 + \sin x}{\sin x}}$$

$$\frac{1 + \sin x}{\cos x} * \frac{\sin x}{1 + \sin x} = \frac{\sin x}{\cos x} = \tan x$$

Sum identity

$\cos 75^\circ$

$\cos 75^\circ = \cos(45^\circ + 30^\circ)$

Sum: $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(45+30) = (\cos 45)(\cos 30) - (\sin 45)(\sin 30)$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{2}$$

$\cos 75^\circ = \cos(120-45)$

Difference Identity

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos 75 = \cos(120-45) = (\cos 120)(\cos 45) + (\sin 120)(\sin 45)$$

$$\left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$-\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2} = \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}$$

$$\boxed{\frac{\sqrt{6}-\sqrt{2}}{2}}$$