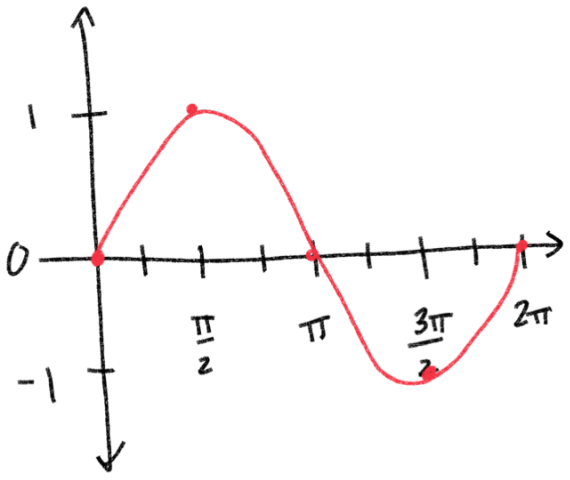
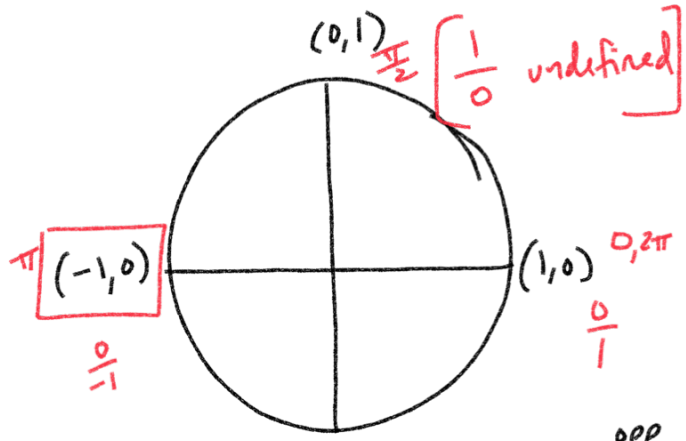
TTH-PT Pre-Calculus: Trigonometry Session 12

$f(x) = \sin x$





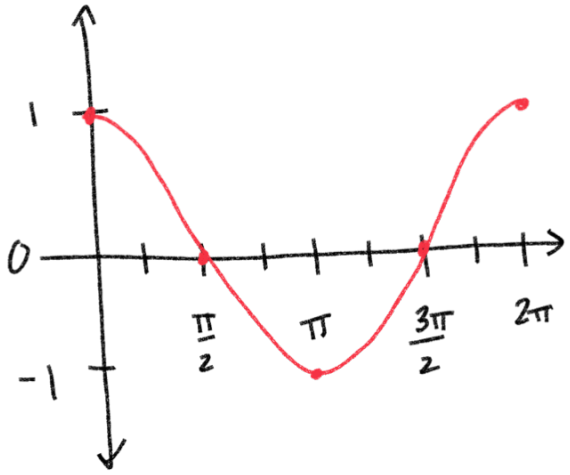
$\sin = \frac{opp}{hyp}$

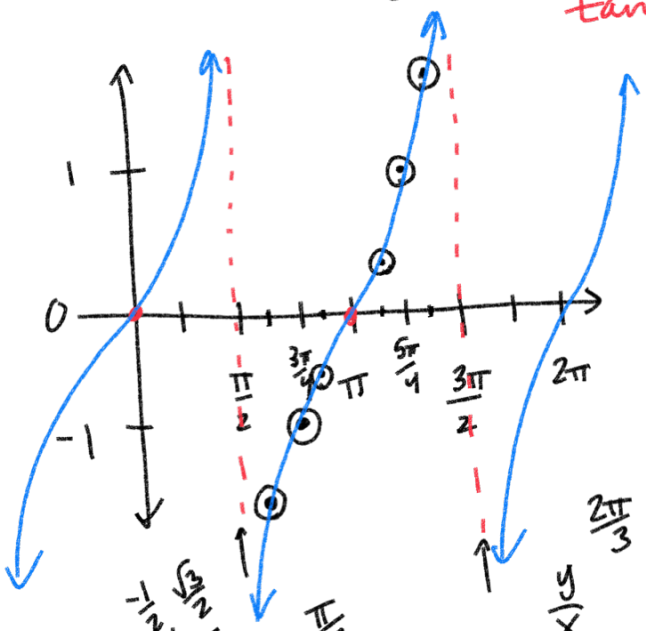
$\cos = \frac{adj}{hyp}$

$f(x) = \tan x = \frac{\sin x}{\cos x}$

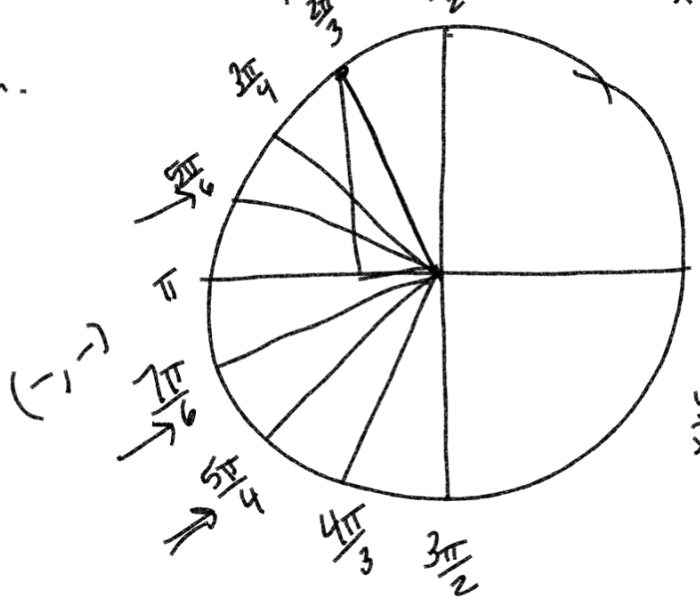
$\tan = \frac{y}{x}$

$f(x) = \cos x$





$\frac{\frac{opp}{hyp}}{\frac{adj}{hyp}} = \frac{opp}{adj} = \tan$



$\frac{y}{x}$

$\frac{\sqrt{3}/2}{-1/2} = -\sqrt{3} = -1.7$

$\frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$

$\frac{1/2}{-\sqrt{3}/2} = \frac{-1}{\sqrt{3}} = \frac{-1}{1.7}$

Even and Odd Functions

$f(-x) = f(x)$

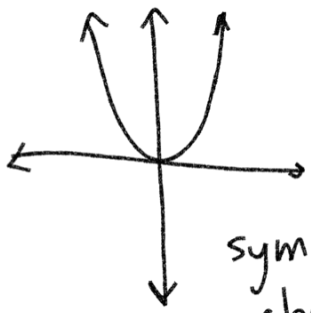
even

$f(x) = x^2$

$f(-2) = (-2)^2 = 4$

$f(2) = (2)^2 = 4$

$f(-2) = f(2)$



symmetry about y-axis

$f(-x) = -f(x)$

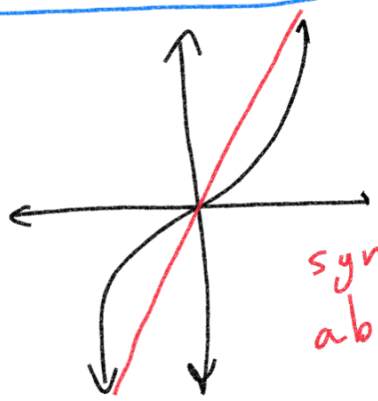
odd

$f(x) = x^3$

$f(-2) = (-2)^3 = -8$

$f(2) = (2)^3 = 8$

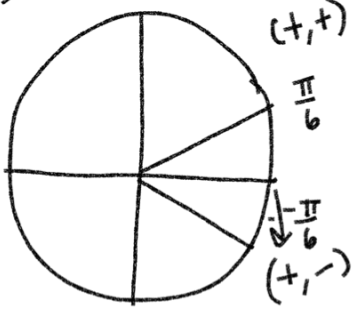
$f(-2) = -f(2)$



symmetry about $x = y$

$\cos(-x) = \cos(x)$

cos → even



$\cos\left(\frac{-\pi}{6}\right) = \cos\frac{\pi}{6}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{6} = \boxed{\frac{\sqrt{3}}{2}}$

$\cos\frac{-\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin(-x) = -\sin(x)$

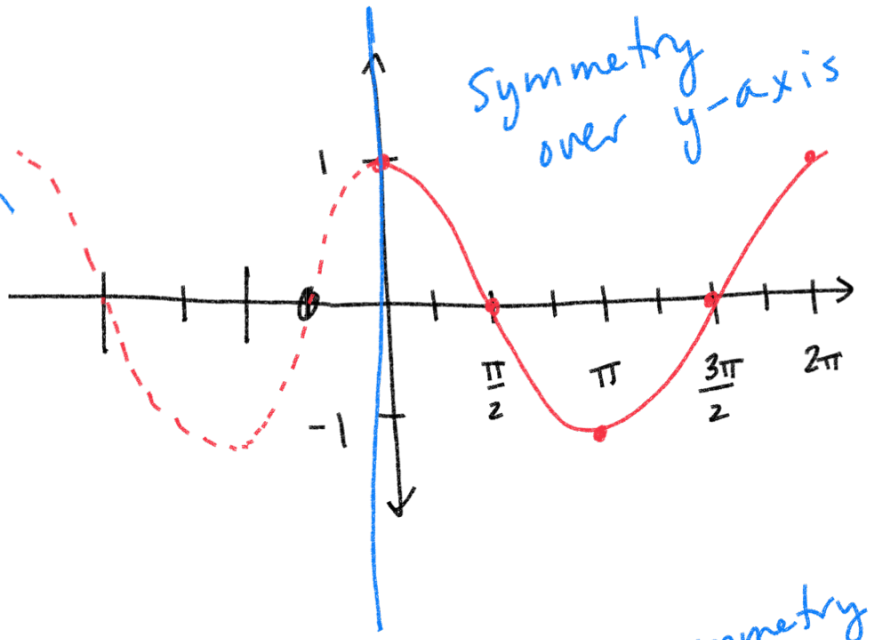
sin → odd

$\sin\frac{\pi}{6} = \boxed{\frac{1}{2}}$

$\sin\frac{-\pi}{6} = -\frac{1}{2}$

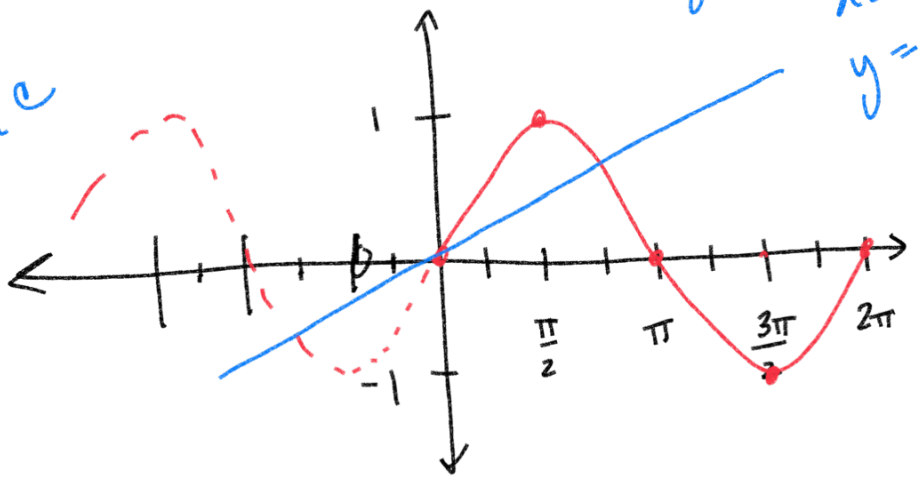
Even function

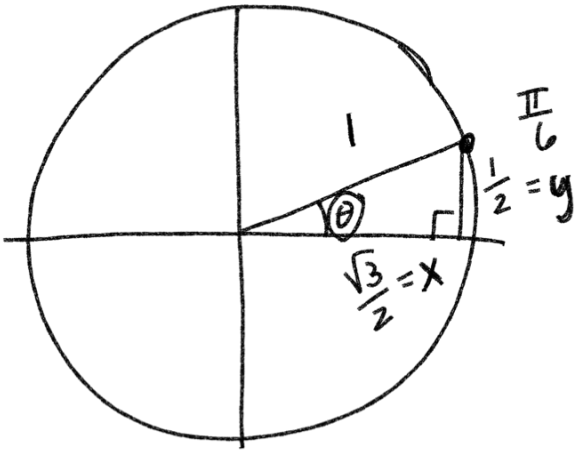
Symmetry over y-axis



odd func

Symmetry about $y = x$





$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1^2$

$\frac{3}{4} + \frac{1}{4} = 1$

$1 = 1$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{1}$

$\sin\theta = y$

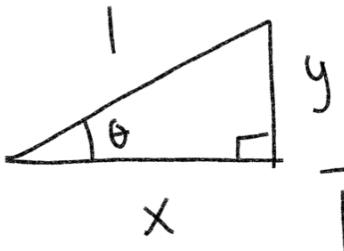
$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{1}$

$\cos\theta = x$

$\sin\frac{\pi}{6} = \frac{1}{2}$ y coordinate

$\sin\theta = y$

$(\cos x)^2 = \cos^2 x$



$x^2 + y^2 = 1$

$$\cos^2 x + \sin^2 x = 1$$

$x = \frac{\pi}{6}$

$\left(\cos\frac{\pi}{6}\right)^2 + \left(\sin\frac{\pi}{6}\right)^2 = 1$

$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$

$\frac{3}{4} + \frac{1}{4} = 1$ ✓

$x = \frac{\pi}{4}$ $\quad \left(\cos\frac{\pi}{4}\right)^2 + \left(\sin\frac{\pi}{4}\right)^2 = 1$

$\Downarrow$

$\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1$

$\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = 1$

$\frac{2}{4} + \frac{2}{4} = 1$ ✓

$x = \pi$ $\quad (\cos\pi)^2 + (\sin\pi)^2 = 1$

$\Downarrow$

$(-1)^2 + (0)^2 = 1$

$1 + 0 = 1$ ✓

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$\frac{1}{\sin x} = \csc x$

$\frac{1}{\sin^2 x} = \csc^2 x$

$\Rightarrow 1 + \cot^2 x = \csc^2 x$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\Rightarrow \tan^2 x + 1 = \sec^2 x$$

Strat: Everything sin/cos

$\sin^2 x + \cos^2 x = 1$

Example 1

$$\tan x \sin x + \cos x = \sec x$$

$$\Downarrow$$

$$\frac{(\sin x)(\sin x)}{\cos x} + \cos x = \frac{1}{\cos x}$$

$$\frac{\cos x}{1}\,\frac{(\cos x)}{\cos x}$$

$$\frac{(\sin x)(\sin x)}{\cos x} + \frac{(\cos x)(\cos x)}{\cos x} = \frac{1}{\cos x}$$

$$\frac{(\sin x)(\sin x) + (\cos x)(\cos x)}{\cos x} = \frac{1}{\cos x}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos x}$$

$\sin^2 x + \cos^2 x = 1$

$$\frac{\sin X}{\sec^2 X} = \frac{\cos^2 X}{\csc X}$$

Keep, Change, Flip

$$\frac{\sin X}{\frac{1}{\cos^2 X}} = \sin X \div \frac{1}{\cos^2 X}$$

$$\sin X * \cos^2 X$$

$$\frac{\sin X}{\frac{1}{\cos^2 X}} = \frac{\cos^2 X}{\frac{1}{\sin X}}$$

$$\sin X \cos^2 X = \sin X \cos^2 X$$

$$(\sin X - \cos X)^2 - 1 = -2 \sin X \cos X$$

FOIL

$$(\sin X - \cos X)(\sin X - \cos X) - 1$$

$$\sin^2 X - \sin X \cos X - \sin X \cos X + \cos^2 X - 1$$

$$\sin^2 X - 2 \sin X \cos X + \cos^2 X - 1 = -2 \sin X \cos X$$

$\sin^2 X + \cos^2 X$

$1 - 1 = 0$

$$\sec x \sin x = \tan x$$

$$\left(\frac{1}{\cos x}\right) \sin x =$$

$$\frac{\sin x}{\cos x} = \tan x$$