Pre-Calculus Chapter 2 Pre-Test

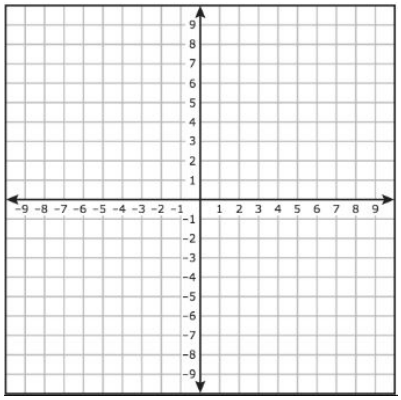
1.) (2.5 pts each, 5 pts total) Determine whether each of the following is a polynomial. If so, identify the degree

a) $f(x) = 2x^5 - 3x^3 + 7x^2 - 9x$

b) $f(x) = 5x^3 + 12x^2 + \sqrt{9x}$

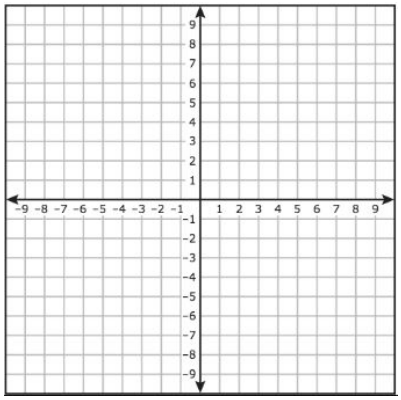
2.) (5 pts) Graph the quadratic function, which is given in standard form

$f(x) = (x + 2)^2 - 4$



3.) (10 pts) Rewrite the quadratic function in standard form by completing the square. Then graph.

$f(x) = 2x^2 + 8x + 5$



4.) (5 pts) Find all of the real zeros (and their state of multiplicities) for the polynomial.

$$f(x) = 6x^2(x - 2)^4(x + 7)^3$$

5.) (10 pts) Find a polynomial of minimum degree that has the given zeros.

-2, 0, 1, 3

6.) (10 pts) For the polynomial function: (a) list each real zero and its multiplicity; (b) determine whether the graph touches or crosses at each x-intercept; (c) find the y-intercept; (d) sketch-ish the graph.

$$f(x) = x^5 - 4x^3$$

$x^3(x^2 - 4) = 0$

$\sqrt[3]{x^3} = \sqrt[3]{0}$

$x = 0$

mult of 3

$x^2 - 4 = 0$

$+4 \quad +4$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$x^3(x-2)(x+2)$

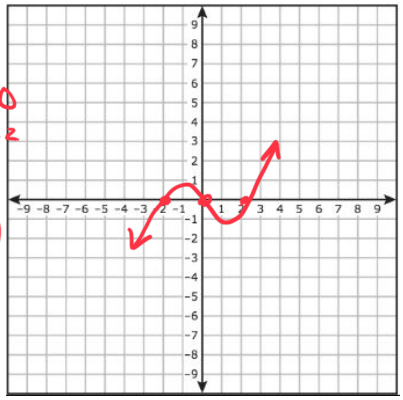
$x - 2 = 0 \quad x + 2 = 0$

$+2 \quad +2 \quad -2 \quad -2$

$x = 2$

$x = -2$

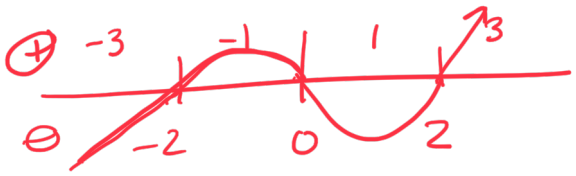
y-int = 0



$(x)^3 (x-2)(x+2)$

-3 — — — (−)

-1 — — + (+)



1 + − + e

3 + + + e

7.) (7.5 pts each, 15 pts total) Divide the polynomials by either long division or synthetic division.

a) $(x^4 - 2x^3 - 7x^2 + 8x + 12) \div (x + 2)$

$x^3 - 4x^2 + x + 6$

$x+2 \;\overline{)\; x^4 - 2x^3 - 7x^2 + 8x + 12}$

$-x^4 \mp 2x^3$ ↓

$-4x^3 - 7x^2$

$+4x^3 + 8x^2$ ↓

$x^2 + 8x$

$-x^2 \mp 2x$

$6x + 12$

$-6x \mp 12$

0

$x + 2 = 0$

$-2 \quad -2$

$x = -2$

$\frac{-4x^3}{x} = -4x^2$

Synthetic

$-2 \;|\; 1 \;\; -2 \;\; -7 \;\; 8 \;\; 12$

$\downarrow \;\; -2 \;\; 8 \;\; -2 \;\; -12$

$1 \;\; -4 \;\; 1 \;\; 6$

$x^3 - 4x^2 + x + 6$

b) $(x^5 + 4x^4 + 3x^2 + 19x + 28) \div (x + 4)$

8.) (10 pts) For the function: $x^4 + 8x^3 + 9x^2 - 38x - 40$

denominators ↓ numerator ↓

a) Find all potential zeros.

40: 1, 2, 4, 5, 8, 10, 20, 40

1: 1, 1

$\pm\frac{5}{1}$ $\pm\frac{40}{1}$ $\pm\frac{10}{1}$ $\pm\frac{20}{1}$ $\pm\frac{8}{1}$ $\pm\frac{1}{1}$ $\pm\frac{4}{1}$ $\pm\frac{2}{1}$

b) Find the number of possible *positive* zeros.

$x^4 + 8x^3 + 9x^2 - 38x - 40$

1

c) Find the number of possible *negative* zeros.

$x^4 + 8x^3 + 9x^2 - 38x - 40$

$x^4 - 8x^3 + 9x^2 + 38x - 40$

① ② ③

3, 1

d) Attempt to find **3 zeros** using long division or synthetic division. Show all work.

1, −1, −2

9.) (10 pts) Find a polynomial of minimum degree with the following zeros:

-4, 3 - *i*, 3 + *i*

$x = -4$ $\quad x = 3 - i$ $\quad x = 3 + i$

$+4 \quad +4 \quad -3+i \quad -3+i \quad -3-i \quad -3-i$

$x + 4 = 0 \quad x - 3 + i = 0 \quad x - 3 - i = 0$

$(x+4) \quad (x-3+i)(x-3-i)$

$x^2 - 3x - ix - 3x + 9 + 3i + ix - 3i - i^2$

$x^2 - 3x - 3x + 9 - i^2$

$-(-1)$

$x^2 - 3x - 3x + 9 + 1$

$i = \sqrt{-1}$

$i^2 = (\sqrt{-1})^2 = -1$

$(x^2 - 6x + 10)$

$(x+4)(x^2 - 6x + 10)$

10.) (10 pts) Given a zero of the polynomial, determine all other zeros (real or complex) and write the polynomial as a product of linear factors.

$x^4 + x^3 - 8x^2 + 4x - 48$, zero = $2i$ $\quad -2i$

$$
\begin{array}{r}
x^2 + x - 12 \\
x^2+0x+4 \,\big)\, x^4 + x^3 - 8x^2 + 4x - 48 \\
-\quad x^4 \quad 0x^3 \mp 4x^2 \\
\hline
x^3 - 12x^2 + 4x \\
-x^3 + 0x^2 - 4x \\
\hline
-12x^2 \quad 0x - 48 \\
+12x^2 \quad +48 \\
\hline
0
\end{array}
$$

$x = 2i$ $\quad -2i \quad -2i$

$x = -2i$ $\quad +2i \quad +2i$

$x - 2i = 0$ $\quad x + 2i = 0$

$(x-2i)(x+2i)$

$x^2 + \cancel{2ix} - \cancel{2ix} - 4i^2$

$i^2 = -1$

$x^2 - 4(-1)$

$x^2 + 4$

$x^2 + x - 12$

$(x+4)(x-3) = 0$

$x = -4, 3$

$2i, -2i$

11.) (5 pts each, 10 pts total) Find the domain and asymptotes (vertical and horizontal) of each of the following rational functions.

a) $\frac{x^2-4}{3x^2-8x+4}$ $= \frac{(x-2)(x+2)}{(x-2)(3x-2)}$ $x=2$ hole

$3x^2-8x+4$

$(x-2)(3x-2)$

$3x-2=0$

$+2 \quad +2$

$\frac{3x}{3} = \frac{2}{3}$

$x = \frac{2}{3}$ $x \neq \frac{2}{3}$ vertical asymptote

$\frac{1x^2}{3x^2}$ $x \to \infty$ $y \to \frac{1}{3}$ horizontal asymptote

b) $\frac{4x^2-3x+6}{8x^3-16x^2+8x}$