Name ______________________ Class ______________ Date ______________

Reteaching 2-1

Conditional Statements

OBJECTIVE: Writing the converse of conditional statements

MATERIALS: None

Example

Write the converse of the following statement.

If snow is falling, then the temperature is below freezing.

If $\underbrace{\text{snow is falling}}_{\text{hypothesis}}$, then $\underbrace{\text{the temperature is below freezing}}_{\text{conclusion}}$.

Converse: Interchange hypothesis and conclusion.
If the temperature is below freezing, then snow is falling.

Exercises

Work in groups of three. Each group member should make up three conditionals relating to sports, hobbies, school, or mathematics.

1. Working alone, write the converse for each conditional.
2. Determine whether each converse is true.
3. Compare your answers with those of the other members of your group. Revise your work until you all agree.

Write the converse for each of the following conditionals. Determine the truth value of each conditional and its converse.

4. If you see lightning, then you hear thunder.
5. If your pants are blue, then they are jeans.
6. If you are eating an orange fruit, then you are eating a tangerine.
7. If a number is a whole number, then it is an integer.
8. If a triangle is an obtuse triangle, then it has one angle greater than 90°.
9. If $n = 8$, then $n^2 = 64$.
10. If you got an A on the first test, then you got an A for the quarter.
11. If a figure is a square, then it has four sides.
12. If $\sqrt{x} = 12$, then $x = 144$.

Name ______________________ Class ______________ Date ______________

Practice 2-1

Conditional Statements

Show that each conditional is false by finding a counterexample.

1. If it is 12:00 noon, then the sun is shining.
2. If the car is full of gas, then the engine will start.
3. If a number is divisible by 3, then it is odd.

Write the converse of each conditional.

4. If you drink milk, then you will be strong.
5. If a rectangle has four sides the same length, then it is a square.
6. If you do not sleep, you will be tired.

Write the converse of each statement. If the converse is true, write *true;* if it is not true, provide a counterexample.

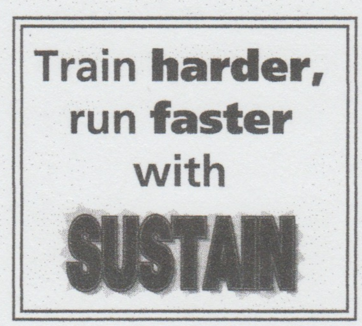
7. If $x - 4 = 22$, then $x = 26$.
8. If $|x| > 0$, then $x > 0$.
9. If m^2 is positive, then m is positive.
10. If $y = 3$, then $2y - 1 = 5$.
11. If point A is in the first quadrant of a coordinate grid, then $x > 0$.
12. If two lines have equal slopes, then the lines are parallel.
13. If you are a twin, then you have a sibling.
14. Draw a Venn diagram to illustrate the statement in Exercise 13.

Answer the following questions about the given quote.

"If you like to shop, then visit the Pigeon Forge outlets in Tennessee."

15. Identify the hypothesis and the conclusion.
16. What does the quote suggest about the Pigeon Forge outlets?
17. Write the converse of the conditional.
18. Is the converse of the conditional a true statement? Explain your reasoning.

Answer the following questions about the billboard advertisement shown.



19. What does the billboard imply?
20. Write the advertisement slogan as a conditional statement.
21. Write the converse of the conditional statement from Exercise 20.

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Reteaching 2-2

Biconditionals and Definitions

OBJECTIVE: Writing biconditional statements and identifying good definitions

MATERIALS: None

Example 1

Consider the true statement given below. Write its converse. If the converse is also true, combine the statements as a biconditional.

Conditional: If a pentagon has five equal sides, then it is an equilateral pentagon.

Converse: If a pentagon is an equilateral pentagon, then it has five equal sides.

The converse is true, so the two statements can be written as one biconditional.

Biconditional: A pentagon is an equilateral pentagon if and only if it has five equal sides.

Example 2

Show that this definition of isosceles triangle is a good definition. Then write it as a true biconditional. *An isosceles triangle has two sides of equal length.*

Conditional: If a triangle has two sides of equal length, then it is an isosceles triangle.

Converse: If a triangle is isosceles, then it has two sides of equal length.

Because the two conditionals are true, this is a good definition and can be rewritten as a biconditional.

Biconditional: A triangle is an isosceles triangle if and only if two sides are of equal length.

Exercises

Write the two conditional statements that make up each biconditional.

1. $|n| = 15$ if and only if $n = 15$ or $n = -15$.
2. Two segments are congruent if and only if they have the same measure.
3. You live in California if and only if you live in the most populated state in the United States.
4. An integer is a multiple of 10 if and only if the last digit is 0.

If the statement is a good definition, write it as a biconditional. If not, find a counterexample.

5. An elephant is a large animal.
6. Two planes intersect at a line.
7. An even number is a number that ends in 0, 2, 4, 6, or 8.
8. A triangle is a three-sided figure whose angle measures sum to 180°.

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Name ______________________ Class ______________ Date ______________

Practice 2-2

Biconditionals and Definitions

Each conditional statement is true. Consider each converse. If the converse is true, combine the statements and write them as a biconditional.

1. If two angles have the same measure, then they are congruent.
2. If $2x - 5 = 11$, then $x = 8$.
3. If $n = 17$, then $|n| = 17$.
4. If a figure has eight sides, then it is an octagon.

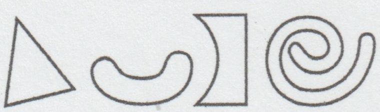
Write the two conditional statements that make up each biconditional.

5. A whole number is a multiple of 5 if and only if its last digit is either a 0 or a 5.
6. Two lines are perpendicular if and only if they intersect to form four right angles.
7. You live in Texas if and only if you live in the largest state in the contiguous United States.

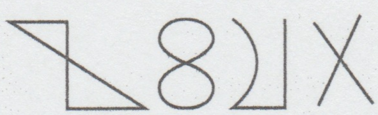
Explain why each of the following is not an acceptable definition.

8. An automobile is a motorized vehicle with four wheels.
9. A circle is a shape that is round.
10. The median of a set of numbers is larger than the smallest number in the set and smaller than the largest number in the set.
11. Cricket is a game played on a large field with a ball and a bat.
12. A rectangle is a very pleasing shape with smooth sides and very rigid corners.

Some figures that are *piggles* are shown below, as are some *nonpiggles*.

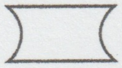


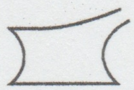
piggles



nonpiggles

Tell whether each of the following is a *piggle*.

13. 

14. 

15. 

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Name ____________________ Class ____________ Date ____________

Reteaching 2-3

Deductive Reasoning

OBJECTIVE: Using the Law of Detachment and the Law of Syllogism to draw conclusions

MATERIALS: None

Example 1

Use the Law of Detachment to draw a conclusion.

If a person goes to the zoo, he or she will see animals. — ***conditional***
Karla goes to the zoo. — ***hypothesis of conditional***

Both the conditional and hypothesis are given to be true. By the Law of Detachment, the conclusion is that Karla will see animals.

However, the Law of Detachment does not apply when a conditional and a *conclusion* are given. Consider the following:

If a person goes to the zoo, he or she will see animals. — ***conditional***
Karla sees animals. — ***conclusion of conditional***

The Law of Detachment cannot be used to say that Karla went to the zoo. In fact, Karla may have seen dogs in the park and not gone to the zoo at all. In this case, no conclusion is possible.

Example 2

Use the Law of Syllogism to draw a conclusion.

If a polygon is a hexagon, then the sum of its angles is 720. — ***conditional 1***
If the sum of the angles of a polygon is 720, then it has six sides. — ***conditional 2***

Both conditionals are given to be true. By the Law of Syllogism, if a polygon is a hexagon, then it has six sides.

Exercises

For each problem, tell which law may be used to draw a conclusion. Then write the conclusion. If a conclusion is not possible, write *not possible* and explain why.

1. If a person is driving over the speed limit, the police officer will give the person a ticket.
   Darlene is driving over the speed limit.
2. If two planes do not intersect, then they are parallel.
   If two planes do not have any points in common, then they do not intersect.
3. If the result of the arm X-ray is positive, then a bone is broken.
   The result of Landon's arm X-ray is positive.
4. If you live in Chicago, then you live in Illinois.
   Brad lives in Illinois.
5. If a figure is a circle, then its circumference is πd.
   Tony draws a circle with a diameter (d) of 1 inch.

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Name ______________________ Class ______________ Date ______________

Practice 2-3

Deductive Reasoning

Use the Law of Detachment to draw a conclusion.

1. If the measures of two angles have a sum of 90°, then the angles are complementary.
   $m\angle A + m\angle B = 90$

2. If the football team wins on Friday night, then practice is canceled for Monday.
   The football team won by 7 points on Friday night.

3. If a triangle has one 90° angle, then the triangle is a right triangle.
   In $\triangle DEF$, $m\angle E = 90$.

Use the Law of Syllogism to draw a conclusion.

4. If you liked the movie, then you saw a good movie.
   If you saw a good movie, then you enjoyed yourself.

5. If two lines are not parallel, then they intersect.
   If two lines intersect, then they intersect at a point.

6. If you vacation at the beach, then you must like the ocean.
   If you like the ocean, then you will like Florida.

If possible, use the Law of Detachment to draw a conclusion. If not possible, write *not possible*.

7. If Robbie wants to save money to buy a car, he must get a part-time job.
   Robbie started a new job yesterday at a grocery store.

8. If a person lives in Omaha, then he or she lives in Nebraska.
   Tamika lives in Omaha.

9. If two figures are congruent, their areas are equal.
   The area of $ABCD$ equals the area of $PQRS$.

Use the Law of Detachment and the Law of Syllogism to draw conclusions from the following statements.

10. If it is raining, the temperature is greater than 32°F.
    If the temperature is greater than 32°F, then it is not freezing outside.
    It is raining.

11. If you live in Providence, then you live in Rhode Island.
    If you live in Rhode Island, then you live in the smallest state in the United States.
    Shannon lives in Providence.

12. If it does not rain, the track team will have practice.
    If the track team has practice, the team members will warm up by jogging two miles.
    It does not rain on Thursday.

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Reteaching 2-4

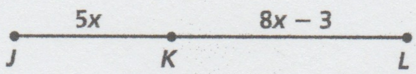
Reasoning in Algebra

OBJECTIVE: Naming and ordering properties used in algebraic reasoning

MATERIALS: None

Example

Use the figure to solve for x. Justify each step.



Given: $JL = 62$

$JK + KL = JL$	Segment Addition Postulate
$5x + (8x - 3) = 62$	Substitution Property
$13x - 3 = 62$	Simplify
$13x = 65$	Addition Property of Equality
$x = 5$	Division Property of Equality

Exercises

Name the properties that justify the steps taken.

1. $AB = EF$; therefore $AB + CD = EF + CD$.

2. $\angle ABC \cong \angle Q$; therefore $\angle Q \cong \angle ABC$.

Support each statement with a reason.

3. $5(y - x) = 20$ Given

$5y - 5x = 20$?

4. $2x = m\angle C + x$ Given

$x = m\angle C$?

5. $CD = AF - 2(CD)$ Given

$3(CD) = AF$?

6. $(q - x) = r$ Given

$4(q - x) = 4r$?

7. $m\angle Q - m\angle R = 90$ Given

$m\angle Q = 4m\angle R$ Given

$4m\angle R - m\angle R = 90$?

8. $m\angle AOX = 2m\angle XOB$ Given

$2m\angle XOB = 140$ Given

$m\angle AOX = 140$?

9. $m\angle P + m\angle Q = 90$ and $m\angle Q = 5m\angle P$. Order the steps given below to show that $m\angle Q = 75$.

1. By the Distributive Property, $6m\angle P = 90$.
2. By substitution, $m\angle Q = 5 \times 15 = 75$.
3. By the Division Property, $m\angle P = 15$.
4. Given: $m\angle P + m\angle Q = 90$, $m\angle Q = 5m\angle P$.
5. By substitution, $m\angle P + 5m\angle P = 90$.

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Name ______________________________ Class ____________________ Date ____________________

Practice 2-4

Use the given property to complete each statement.

1. Symmetric Property of Equality
   If $MN = UT$, then ? .

2. Division Property of Equality
   If $4m\angle QWR = 120$, then ? .

3. Transitive Property of Equality
   If $SB = VT$ and $VT = MN$, then ? .

4. Addition Property of Equality
   If $y - 15 = 36$, then ? .

5. Reflexive Property of Congruence
   $\overline{JL} \cong$?

Give a reason for each step.

6. $7x - 4 = 10$
   $7x = 14$
   $x = 2$

7. $0.25x + 2x + 12 = 39$
   $2.25x + 12 = 39$
   $2.25x = 27$
   $225x = 2700$
   $x = 12$

Name the property that justifies each statement.

8. If $m\angle G = 35$ and $m\angle S = 35$, then $m\angle G \cong m\angle S$.

9. If $10x + 6y = 14$ and $x = 2y$, then $10(2y) + 6y = 14$.

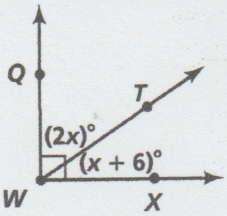
10. If $TR = MN$ and $MN = VW$, then $TR = VW$.

11. If $\overline{JK} \cong \overline{LM}$, then $\overline{LM} \cong \overline{JK}$.

12. If $\angle Q \cong \angle S$ and $\angle S \cong \angle P$, then $\angle Q \cong \angle P$.

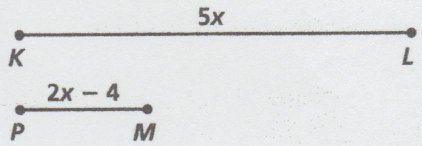
Fill in the missing information. Solve for x, and justify each step.

13.



$m\angle QWT + m\angle TWX = 90$
$2x + (x + 6) =$?
? $+ 6 = 90$
? = ?
$x =$?

14.



$KL = 3(PM)$
$5x = 3$?
$5x =$?
? $= -12$
$x =$?

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Reteaching 2-5

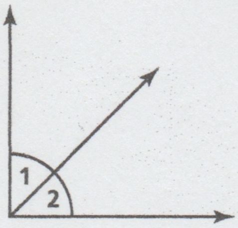
Proving Angles Congruent

OBJECTIVE: Using deductive reasoning to solve problems and verify conjectures

MATERIALS: None

Example

Suppose that two complementary angles are congruent. Prove that the measure of each angle is 45.



Given: $\angle 1$ and $\angle 2$ are complementary.

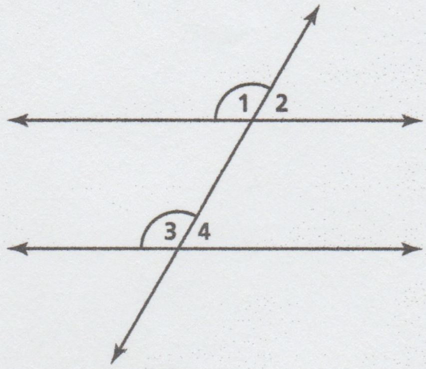
$m\angle 1 = m\angle 2$

Prove: $m\angle 1 = 45$ and $m\angle 2 = 45$

By the definition of complementary angles, $m\angle 1 + m\angle 2 = 90$. By substitution, $m\angle 1 + m\angle 1 = 90$. Using the Addition Property of Equality, $2m\angle 1 = 90$. Using the Division Property of Equality, $m\angle 1 = 45$. By substitution, $m\angle 2 = 45$.

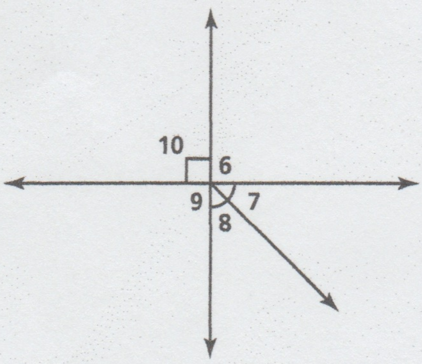
Exercises

In the diagram, $m\angle 1 = m\angle 3$. Order the steps given below to prove that $m\angle 2 = m\angle 4$.



1. By the Angle Addition Postulate, $m\angle 3 + m\angle 4 = 180$.
2. Prove: $m\angle 2 = m\angle 4$
3. By substitution, $m\angle 3 + m\angle 2 = m\angle 3 + m\angle 4$.
4. By the Angle Addition Postulate, $m\angle 1 + m\angle 2 = 180$.
5. Given: $m\angle 1 = m\angle 3$
6. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$ by the Transitive Property of Equality.
7. Subtract $m\angle 3$ from both sides, and you get $m\angle 2 = m\angle 4$.

In the diagram, $\angle 7$ and $\angle 8$ are congruent, and $\angle 10$ is a right angle. Explain why each statement is true.



8. $m\angle 8 = 45$
9. $\angle 9$ and $\angle 10$ are supplementary.
10. $\angle 6$ is a right angle.

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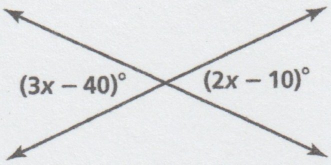
Name ____________________ Class ____________ Date ____________

Practice 2-5

Proving Angles Congruent

Find the values of the variables.

1.



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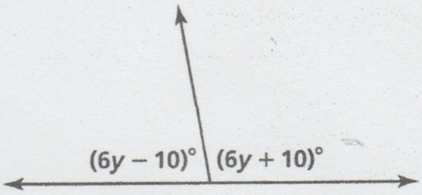

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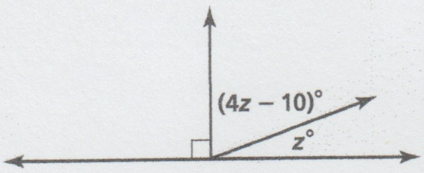

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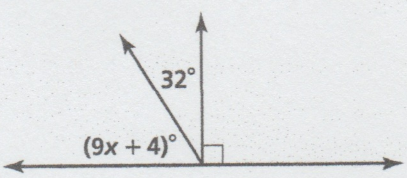

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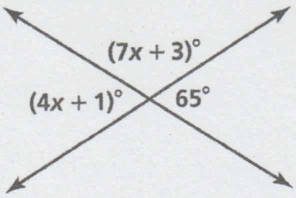

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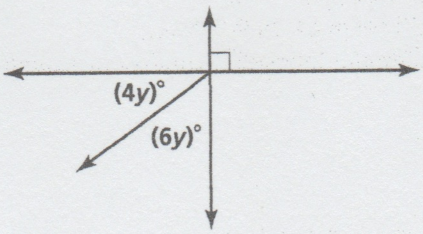


Write *true* or *false*.

7. $\angle 1$ and $\angle 2$ are vertical angles.

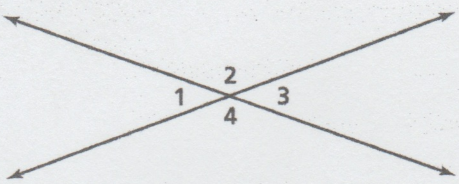
8. $\angle 2$ and $\angle 3$ are supplementary angles.

9. $m\angle 1 = m\angle 3$

10. $m\angle 3 + m\angle 4 = 180$

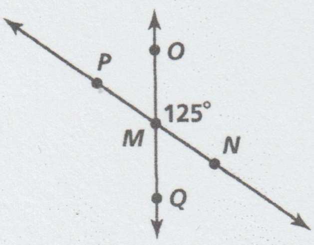
11. $m\angle 1 + m\angle 3 = 180$

12. $\angle 4$ and $\angle 2$ are adjacent angles.

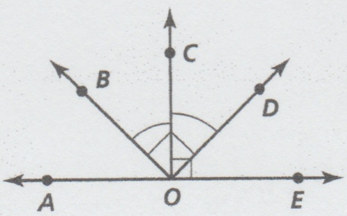


Write three conclusions that can be drawn from each figure.

13.



14.



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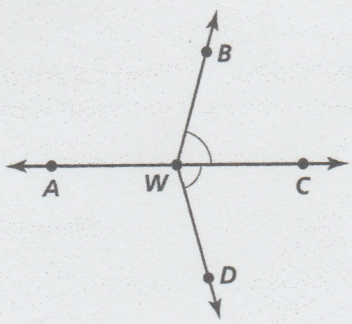

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Algebraic Properties and Proofs

Name ________________________

You have solved algebraic equations for a couple years now, but now it is time to justify the steps you have practiced and now take without thinking… and acting without thinking is a dangerous habit!

The following is a list of the reasons one can give for each algebraic step one may take.

ALGEBRAIC PROPERTIES OF EQUALITY	
ADDITION PROPERTY OF EQUALITY	If $a = b$, then $a + c = b + c$
SUBTRACTION PROPERTY OF EQUALITY	If $a = b$, then $a - c = b - c$
MULTIPLICATION PROPERTY OF EQUALITY	If $a = b$, then $a \cdot c = b \cdot c$
DIVISION PROPERTY OF EQUALITY	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$
DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION or OVER SUBTRACTION	$a(b + c) = ab + ac$ $a(b - c) = ab - ac$
SUBSTITUTION PROPERTY OF EQUALITY	If $a = b$, then b can be substituted for a in any equation or expression
REFLEXIVE PROPERTY OF EQUALITY	For any real number a, $a = a$
SYMMETRIC PROPERTY OF EQUALITY	If $a = b$, then $b = a$
TRANSITIVE PROPERTY OF EQUALITY	If $a = b$ and $b = c$, then $a = c$

Complete the following algebraic proofs using the reasons above. If a step requires simplification by combining like terms, write ***simplify***.

Given: $3x + 12 = 8x - 18$

Prove: $x = 6$

Statements	Reasons
1. $3x + 12 = 8x - 18$	1.
2. $12 = 5x - 18$	2.
3. $30 = 5x$	3.
4. $6 = x$	4.
5. $x = 6$	5.

Given: $3k + 5 = 17$
Prove: $k = 4$

Statements	Reasons
1. $3k + 5 = 17$	1.
2. $3k = 12$	2.
3. $k = 4$	3.

Given: $-6a - 5 = -95$
Prove: $a = 15$

Statements	Reasons

Given: $3(5x + 1) = 13x + 5$
Prove: $x = 1$

Statements	Reasons

Given: $7y - 84 = 2y + 61$

Prove: $y = 29$

Statements	Reasons

Given: $4(5n + 7) - 3n = 3(4n - 9)$

Prove: $n = -11$

Statements	Reasons

Given: $4.7\quad 2f - 0.5\quad = -6\quad 1.6f - 8.3f$

Prove: $y = -\dfrac{47}{616}$

Statements	Reasons

Geometric Properties

We have discussed the RST (Reflexive, Symmetric, and Transitive) properties of equality. We could prove that these also apply for congruence… but we won't. We are just going to accept it…

I know, you're disappointed.

PROPERTIES OF CONGRUENCE	
REFLEXIVE PROPERTY OF CONGRUENCE	For any geometric figure A, $A \cong A$.
SYMMETRIC PROPERTY OF CONGRUENCE	If $A \cong B$, then $B \cong A$.
TRANSITIVE PROPERTY OF CONGRUENCE	If $A \cong B$ and $B \cong C$, then $A \cong C$
Additional Reasons for Proofs	
DEFINITIONS	
POSTULATES	
PREVIOUSLY PROVED THEOREMS	
ALGEBRAIC PROPERTIES	

Elementary Geometric Proofs

Using Definitions

Given: $\overline{XY} \cong \overline{BC}$

Prove: $XY = BC$

Statements	Reasons

Given: $\angle A \cong \angle Z$

Prove: $m\angle A = m\angle Z$

Statements	Reasons

Using the Transitive Property and Substitution

Given: $m\angle 1 = 45^\circ$; $m\angle 2 = m\angle 1$

Prove: $m\angle 2 = 45^\circ$

Statements	Reasons

You should be aware that there are many ways to complete a proof. In fact, the following website has 79 distinct proofs for the most famous of all theorems, the Pythagorean Theorem.

http://www.cut-the-knot.org/pythagoras/index.shtml

Even the simple proof above could be done in at least two ways. The last statement could have been justified using SUBSTITUTION or the TRANSITIVE PROPERTY. These properties are similar, but no the same:

SUBSTITUTION works only on NUMBERS (=), while the TRANSITIVE PROPERTY can be used to describe relationships between FIGURES or NUMBERS (= or $\cong$). Keep this in mind.

Given: $\angle 1 \cong \angle 2$; $\angle 1 \cong \angle 3$

Prove: $\angle 2 \cong \angle 3$

Statements	Reasons

Using Multiple Reasons

Given: $m\angle A = 90^\circ$; $\angle A \cong \angle Z$

Prove: $\angle Z$ is a right angle

Statements	Reasons

Given: $m\angle 1 = 90^\circ$; $\angle 1 \cong \angle 2$; $\angle 2 \cong \angle 3$

Prove: $\angle 3$ is a right angle

Statements	Reasons

Given: $m\angle O = 180^\circ$; $m\angle P = m\angle S$; $\angle O \cong \angle P$

Prove: $\angle S$ is a straight angle

Statements	Reasons

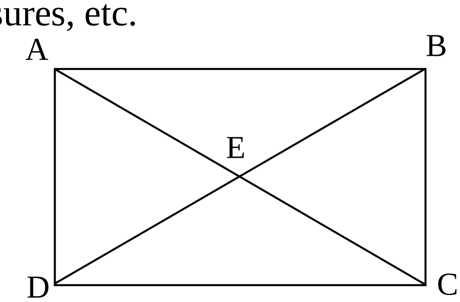
DEFINITIONS AND POSTULATES REGARDING SEGMENTS	
SEGMENT ADDITION POSTULATE	If *C* is between *A* and *B*, then AC + CB = AB
DEFINITION OF SEGMENT CONGRUENCE	If $\overline{AB} \cong \overline{CD}$, then AB = CD
DEFINITION OF A SEGMENT BISECTOR	*A geometric figure that divides a segment in to two congruent halves*
DEFINITION OF A MIDPOINT	*A point that bisects a segment*
DEFINITIONS AND POSTULATES REGARDING ANGLES	
ANGLE ADDITION POSTULATE	If *C* is on the interior of $\angle ABD$, then $m\angle ABC + m\angle CBD = m\angle ABD$
DEFINITION OF ANGLE CONGRUENCE	If $\angle A \cong \angle B$, then $m\angle A = m\angle B$
DEFINITION OF AN ANGLE BISECTOR	*A geometric figure that divides a angle in to two congruent halves*

Proofs with Pictures

It is often much easier to plan and finish a proof if there is a visual aid. Use the picture to help you plan and finish the proof. Be sure that as you write each statement, you make the picture match your proof by inserting marks, measures, etc.

Given: E is the midpoint of $\overline{AC}$ and $\overline{BD}$; $\overline{ED} \cong \overline{EC}$

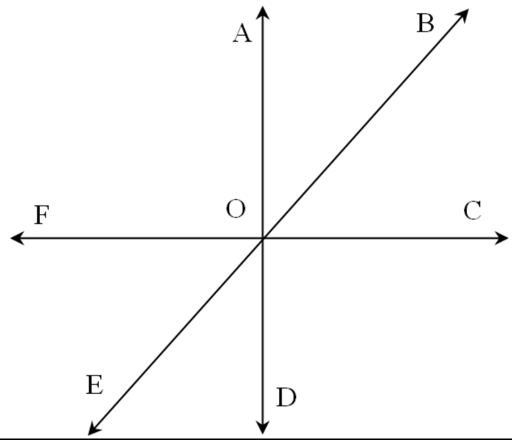
Prove: $\overline{AE} \cong \overline{BE}$



Statements	Reasons

Given: $\overrightarrow{OB}$ bisects $\angle AOC$;
$\overrightarrow{OE}$ bisects $\angle DOF$;
$\angle AOB \cong \angle DOE$

Prove: $\angle EOF \cong \angle BOC$



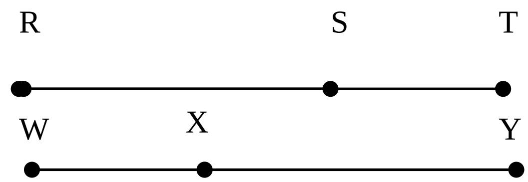
Statements	Reasons

Elementary Geometric Proofs

Segments

Given: $\overline{RT} \cong \overline{WY}$; $\overline{ST} \cong \overline{WX}$

Prove: $\overline{RS} \cong \overline{XY}$

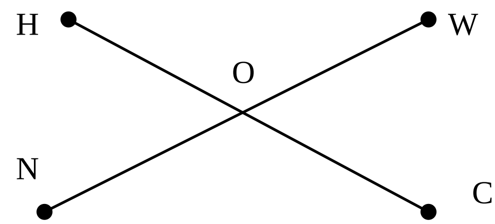


Statements	Reasons

Given: O is the midpoint of $\overline{NW}$;

$\overline{NO} \cong \overline{OC}$

Prove: $\overline{OC} \cong \overline{OW}$



Statements	Reasons
1. O is the midpoint of $\overline{NW}$	1.
2. $\overline{NO} \cong \overline{OW}$	2.
3.	3. Given
4. $\overline{OC} \cong \overline{OW}$	4.

Given: $\overline{EF} \cong \overline{GH}$

Prove: $\overline{EG} \cong \overline{FH}$

Statements	Reasons
1. $\overline{EF} \cong \overline{GH}$	1.
2. EF = GH	2.
3. EF + FG = GH + FG	3.
4. EF + FG = EG; GH + FG = FH	4.
5. EG = FH	5.
6. $\overline{EG} \cong \overline{FH}$	6.

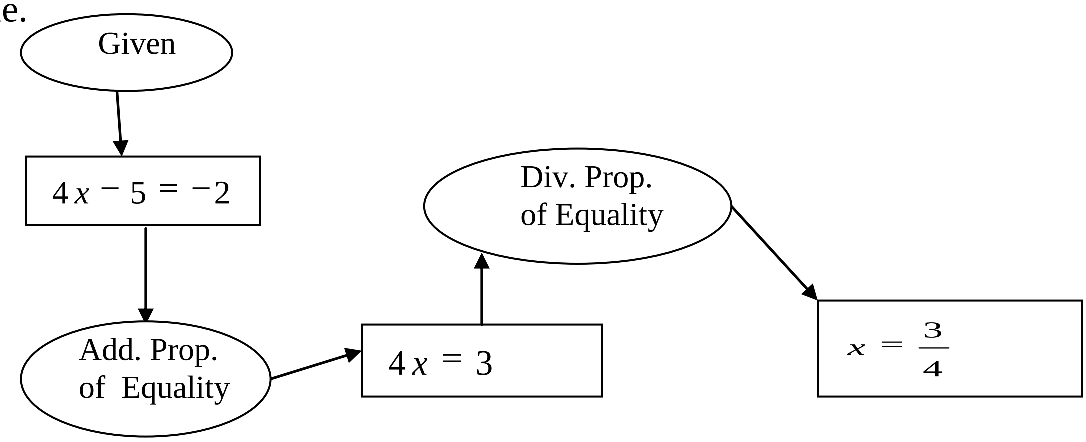
Flow Proofs

Proofs do not always come in two-column format. Sometimes they are more visual, as you will see in this example.

Flow Proof

Given: $4x - 5 = -2$

Prove: $x = \frac{3}{4}$



Complete the flow chart for the following proof.

Given: $AC = CE$; $AB = DE$

Prove: C is the midpoint of $\overline{BD}$