Key

Pre-Calculus Chapter 0.5 Practice Test

1.) (8 pts tot, 4 pts each) Calculate the distance between the given points.

a) (-4, 5) and (-9, -7)

$x_1\ y_1 \qquad x_2\ y_2$

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(-9-(-4))^2 + (-7-5)^2}$$

$$= \sqrt{(-9+4)^2 + (-12)^2}$$

$$= \sqrt{(-5)^2 + (-12)^2}$$

$$\sqrt{25 + 144}$$

$$\sqrt{169}$$

$$\boxed{13}$$

b) (0, -7) and (-4, -5)

$x_1\ y_1 \qquad x_2\ y_2$

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(-4-0)^2 + (-5-(-7))^2}$$

$$\sqrt{(-4)^2 + (-5+7)^2}$$

$$\sqrt{16 + (2)^2}$$

$$\sqrt{16+4}$$

$$\sqrt{20}$$

$$\boxed{2\sqrt{5}}$$

2.) (8 pts tot, 4 pts each) Find the midpoint of the segment joining the two points.

a) (-3, -1) and (-7, 2)

$x_1\ y_1 \qquad x_2\ y_2$

$$\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$$

$$\left(\frac{-7+(-3)}{2}, \frac{2+(-1)}{2}\right)$$

$$\left(\frac{-10}{2}, \frac{1}{2}\right)$$

$$\boxed{\left(-5, \frac{1}{2}\right)}$$

b) (-5, 12) and (7, 16)

$x_1\ y_1 \qquad x_2\ y_2$

$$\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$$

$$\left(\frac{7+(-5)}{2}, \frac{16+12}{2}\right)$$

$$\left(\frac{2}{2}, \frac{28}{2}\right)$$

$$\boxed{(1, 14)}$$

3.) (8 pts tot, 4 pts each) Find the x- and y-intercepts and graph the corresponding lines.

a) $y = -3x + 2$

x-intercept $x = \frac{2}{3}$

$y = 0$

$0 = -3x + 2$

$-2 \quad -2$

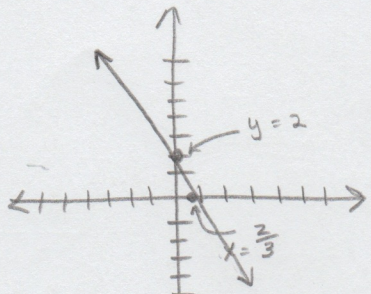
$\frac{-2}{-3} = \frac{-3x}{-3}$

y-intercept

$x = 0$

$y = -3(0) + 2$

$y = 2$



b) $y = x^2 + 6x - 27$

$0 = x^2 + 6x - 27$

$x^2 + 6x = 27$

$x^2 + 6x + 9 = 36$

$(x+3)^2 = 36$

$(x+3)^2 - 36 = 0$

vertex: $(-3, -36)$

x-intercept

$y = 0$

$0 = x^2 + 6x - 27$

$0 = (x+9)(x-3)$

$(x+9) = 0 \quad (x-3) = 0$

$-9 \quad -9 \quad +3 \quad +3$

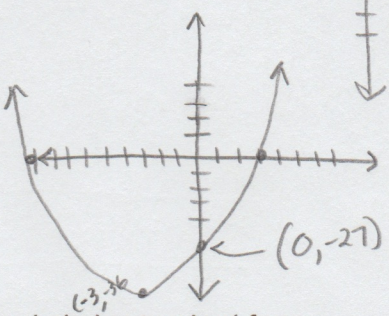
$x = -9 \quad x = 3$

y-intercept

$x = 0$

$y = (0)^2 + 6(0) - 27$

$y = -27$



4.) (8 pts tot, 4 pts each) Write the equation of the circle in standard form.

a) Center (6, -7)
r = 8

$(x-h)^2 + (y-k)^2 = r^2$ $\quad (h,k) = \text{center}$

$\boxed{(x-6)^2 + (y+7)^2 = 64}$

b) Center (-4, -1)
$r = 3\sqrt{5}$

$(x-h)^2 + (y-k)^2 = r^2$

$(x+4)^2 + (y+1)^2 = (3\sqrt{5})^2 = 9 \cdot 5 = 45$

$\boxed{(x+4)^2 + (y+1)^2 = 45}$

5.) (8 pts tot, 4 pts each) State the center and radius of the circle with the given equation.

a) $(x + 3)^2 + (y - 7)^2 = 81$

$(x-h)^2 + (y-k)^2 = r^2$

$(h, k) = (-3, 7)$ $\sqrt{81} = r$

radius = 9

b) $(x + 1)^2 + (y + 2)^2 = 8$

$(x-h)^2 + (y-k)^2 = r^2$

center: $(-1, -2)$

radius $= \sqrt{8} = 2\sqrt{2}$

6.) (8 pts tot, 4 pts each) Find the center and radius of the circle.

a) $x^2 + y^2 + 8x + 2y - 28 = 0$

$x^2 + 8x + 16$ $y^2 + 2y + 1$ $= 28 + 16 + 1 = 45$

$\left(\frac{8}{2}\right)^2$ $\left(\frac{2}{2}\right)^2$

$(x + 4)^2 + (y + 1)^2 = 45$

center $(-4, -1)$ radius $= \sqrt{45} = 3\sqrt{5}$

b) $x^2 + y^2 - 2x - 10y + 2 = 0$

$x^2 - 2x + 1$ $y^2 - 10y + 25 =$ $-2 + 1 + 25$

$\left(\frac{-2}{2}\right)^2$ $\left(\frac{-10}{2}\right)^2$

$(x - 1)^2 + (y - 5)^2 = 24$

center: $(1, 5)$

radius: $\sqrt{24} = 2\sqrt{6}$

7.) (8 pts tot, 4 pts each) Find the slope of the line that passes through the given point.

$x_1\ y_1 \qquad x_2\ y_2$

a) (11, -3) and (-2, 6)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6-(-3)}{-2-11} = \frac{-9}{13}$$

b) (-1, -4) and (4, 6)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{6-(-4)}{4-(-1)} = \frac{6+4}{4+1} = \frac{10}{5} = 2$$

8.) (8 pts tot, 4 pts each) Write the equation in slope-intercept form. Identify the slope and the y-intercept.

slope-intercept form = $y = mx + b$

a) $3x - 5y = 15$

$+5y \quad +5y$

$3x = 5y + 15$

$-15 \quad -15$

$\frac{5y}{5} = \frac{3x}{5} - \frac{15}{5}$

$\boxed{y = \frac{3}{5}x - 3}$

b) $8 = 4x - 16y$

$+16y \quad +16y$

$16y + 8 = 4x$

$-8 \quad -8$

$\frac{16y}{16} = \frac{4x}{16} - \frac{8}{16}$

$\boxed{y = \frac{x}{4} - \frac{1}{2}}$

9.) (8 pts tot, 4 pts each) Write the equation of the line in both point-slope and slope-intercept form.

a) Slope: m = -6 y-intercept: (0, 9)

$y - 9 = -6(x - 0)$

$y - 9 = -6x$

$+9 \quad +9$

Point-slope: $y - y_1 = m(x - x_1)$

$y - 9 = -6(x - 0)$

slope-intercept: $y = mx + b$

$y = -6x + 9$

b) Slope: m = 0 y-intercept: (0, -4)

$y + 4 = 0(x - 0)$

$y = -4$

$y + 4 = 0$

$y = -4$

10.) (8 pts tot, 4 pts each) Write the equation of the line that passes through the given point. Express the equation in slope-intercept form.

a) Slope: m = -⅓
(-6, 9)

$y - y_1 = m(x - x_1)$

$y - 9 = -\frac{1}{3}(x + 6)$

$y = -\frac{1}{3}x + 7$

$y - 9 = -\frac{1}{3}x - 2$

$+9 \quad +9$

b) Slope: m = 4
(-2, 8)

$y - 8 = 4(x + 2)$

$y - 8 = 4x + 8$

$+8 \quad +8$

$y = 4x + 16$

11.) (8 pts tot, 4 pts each) Find the equation of the line that passes through the given point and also satisfies the additional piece of information.

a) (1, 4); perpendicular to $6x + 14y = 7$

slope $= -\left(\frac{A}{B}\right) = -\left(\frac{6}{14}\right) = \frac{-3}{7}$

perpendicular lines
opposite inverse
$-\left(\frac{7}{-3}\right) = \frac{7}{3}$

$y - y_1 = m(x - x_1)$

$\boxed{y - 4 = \frac{7}{3}(x - 1)}$

b) (3, 5); parallel to $3x - 8y = 20$

slope $= -\left(\frac{A}{B}\right) = -\left(\frac{3}{-8}\right) = \frac{3}{8}$

parallel lines have same slopes.

$y - y_1 = m(x - x_1)$

$\boxed{y - 5 = \frac{3}{8}(x - 3)}$

12.) (4 pts each) Write an equation that describes the variation.

a) P varies inversely with r^2

$P = \frac{k}{r^2}$

13.) (8 pts tot, 4 pts each) Write an equation that describes the variation.

a) y varies inversely with both x and z; y = 32, x = 4, z = 0.05

$y = \frac{k}{xz}$ $\quad 32 = \frac{k}{(4)(0.05)}$

$0.2(32) = \left(\frac{k}{0.2}\right)0.2$ $\quad k = 6.4$

$\boxed{y = \frac{6.4}{xz}}$

b) V varies directly with h; V = 18, h = 8

$V = kh$

$18 = k(8)$ $\quad \frac{18}{8} = \frac{8k}{8}$

$\frac{9}{4} = k$

$\boxed{V = \frac{9h}{4}}$