Pre-Calculus Chapter 3 Pre-Test

Key

1.) (2 pts each, 6 pts total) Evaluate exactly. Do not use decimals.

a) 7^{-2} $\frac{1}{7^2}$

b) $8^{2/3}$ $\left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$

c) $(1/4)^{5/2}$ $\left(\sqrt{\frac{1}{4}}\right)^5 = \left(\frac{1}{2}\right)^5 = \frac{1^5}{2^5} = \boxed{\frac{1}{32}}$

2.) (2 pts each, 4 pts total) Evaluate each function.

a) $f(x) = 4^x$, $x = 3$ $f(3) = 4^3 = \boxed{64}$

b) $g(x) = 10^{x+4}$, $x = -2$ $g(-2) = 10^{-2+4} = 10^2 = \boxed{100}$

3.) (5 pts each, 10 pts total) Graph each function. Identify at least two points on the line (please use points indicated in class).

a) $y = 3^{x+2}$

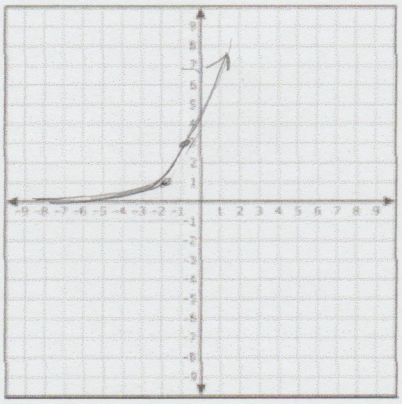
3^a find when $a = 0$ and $a = 1$

$x + 2 = 0$ $\quad x = -2$
$-2 \quad -2$

$x + 2 = 1$ $\quad x = -1$
$-2 \quad -2$

$x = -2$ $\quad y = 3^{-2+2} = 3^0 = 1 \quad (-2, 1)$

$x = -1$ $\quad y = 3^{-1+2} = 3^1 = 3 \quad (-1, 3)$



b) $y = 5^{x-3} + 2$

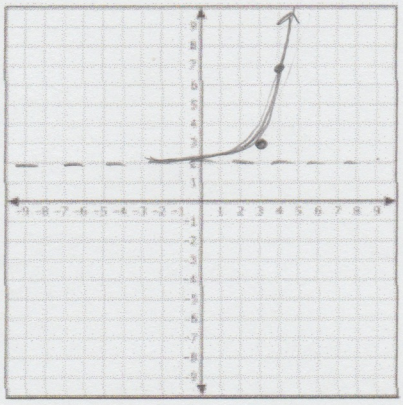
5^a find when $a = 0$ and $a = 1$

$x - 3 = 0$
$+3 \quad +3$
$x = 3$

$y = 5^{3-3} + 2 \quad (3, 3)$
$5^0 + 2 = 1 + 2 = 3$

$x - 3 = 1$
$+3 \quad +3$
$x = 4$

$y = 5^{4-3} + 2$
$5^1 + 2 = 7 \quad (4, 7)$



4.) (2 pts each, 4 pts total) Write each logarithmic equation in its equivalent exponential form.

a) $\log_6 216 = 3$

$6^3 = 216$

b) $\log_b x = a$

$b^a = x$

5.) (2 pts each, 4 pts total) Write each exponential equation in its equivalent logarithmic form.

logs are exponents

a) $4^7 = 16384$ $\log_4 16384 = 7$

b) $0.001 = 10^{-3}$ $\log_{10} 0.001 = -3$ or $\log 0.001 = -3$

6.) (2 pts each, 4 pts total) Evaluate the logarithms exactly. Show conversion to exponential form for full credit.

a) $\log_8 1$ $\log_8 1 = x$ $8^x = 1$ $\boxed{x = 0}$

b) $\log 10^{-5}$ $\log_{10} 10^{-5} = x$ $10^x = 10^{-5}$ $\boxed{x = -5}$

7.) (8 pts total) State the domain of the logarithmic function. Please show work (do not simply graph).

$$f(x) = \log_3 (x - 2)$$

Even if $a \to -\infty$ $\log_3 (x-2) = a$

$3^{-\infty} = \frac{1}{3^{\infty}} = 0$ $3^a = x - 2$

3^a must be greater than 0 therefore $x - 2 > 0$, $+2$ $+2$ $\boxed{x > 2}$

8.) (2.5 pts each, 5 pts total) Apply the properties of logarithms to simplify each expression.

a) $8^{3\log_8 5}$

$8^{3\log_8 5} = x$

$\log_8 x = 3\log_8 5$

$\log_8 x = \log_8 5^3$ → $x = 5^3 = 125$

b) $e^{\ln(x^2-4)} = 2$

$e^{\ln(x^2-4)} = a$

$\ln 2 = \ln(x^2-4)$

$x^2 - 4 = 2$

$+4 \quad +4$

$\sqrt{x^2} = \sqrt{6}$

$x = \pm\sqrt{6}$

9.) (5 pts each, 10 pts total) Write each expression as a sum or difference of logarithms.

a) $\log_b\left(\frac{x^2y^7z^{-3}}{a^4}\right)$

$2\log_b x + 7\log_b y - 3\log_b z - 4\log a$

b) $\log_b\left(\frac{x^2+2x-3}{x^2-6x+8}\right)$

$\log_b(x^2+2x-3) - \log_b(x^2-6x+8)$

$\log_b(x+3)(x-1) - \log_b(x-4)(x-2)$

$\log_b(x+3) + \log_b(x-1) - \log_b(x-4) - \log_b(x-2)$

10.) (5 pts each, 10 pts total) Write each expression as a single logarithm.

a) $6\log_b a + 2\log_b c - 3\log_b d$

$\log_b\left(\frac{a^6c^2}{d^3}\right)$

b) $\frac{1}{2}\log e - 3\log f - 2\log h$

$\frac{e^{\frac{1}{2}}}{f^3h^2}$ or $\frac{\sqrt{e}}{f^3h^2}$

11.) (5 pts each, 10 pts total) Evaluate each logarithm using change-of-base formula.

a) $\log_9 23$

$\log_9 23 = x$

$9^x = 23$

$\log 9^x = \log 23$

$\frac{x \log 9}{\log 9} = \frac{\log 23}{\log 9}$

$\boxed{x = \frac{\log 23}{\log 9}}$

b) $\log_2 a$

$\log_2 a = x$

$2^x = a$

$\log 2^x = \log a$

$\frac{x \log 2}{\log 2} = \frac{\log a}{\log 2}$

$\boxed{x = \frac{\log a}{\log 2}}$

12.) (5 pts each, 15 pts total) Solve each exponential equation. Leave answers as a fraction if necessary (no decimals).

a) $\log (3x + 4) = 2$

$\log_{10} (3x+4) = 2$

$10^2 = 3x+4$

$100 = 3x+4$

$-4 \quad -4$

$\frac{96}{3} = \frac{3x}{3}$

$\boxed{x = 32}$

b) $\log_2 (x - 2) + \log_2 (x + 4) = 4$

$\log_2 (x-2)(x+4) = 4$

$2^4 = (x-2)(x+4)$

$2^4 = x^2 + 2x - 8$

$16 = x^2 + 2x - 8$

$-16 \quad -16$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x+6 = 0 \quad x-4 = 0$

$-6 \quad -6 \quad +4 \quad +4$

$\boxed{x = -6 \quad x = 4}$

c) $\ln(x) + \ln(x+2) - \ln(3x) = 6$

$\ln \frac{x(x+2)}{3x} = 6$

$\ln \frac{x+2}{3} = 6$

$e^6 = \frac{x+2}{3}$

$3e^6 = x + 2$

$-2 \quad -2$

$x = 3e^6 - 2$

13.) (1 pts each, 10 pts total) Complete the table by including the corresponding letter.

Graph Name	Model	Graph
Gaussian distribution	c	h
Logistic growth	d	g
Exponential growth	a	j
Logarithmic	e	f
Exponential decay	b	i

a) $f(t) = ce^{kt}$ $k > 0$

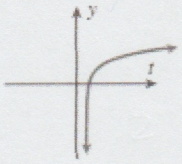
b) $f(t) = ce^{-kt}$ $k > 0$

c) $f(x) = ce^{\frac{-(x-a)^2}{k}}$

d) $f(t) = \frac{a}{1 + ce^{-kt}}$

e) $f(t) = a + c \log t$

f)



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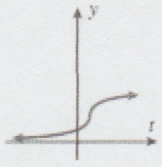

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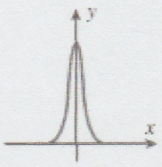

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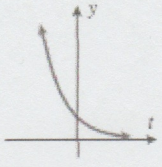

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