Key

Pre-Calculus Chapter 2 Pre-Test

1.) (2.5 pts each, 5 pts total) Determine whether each of the following is a polynomial. If so, identify the degree

a) $f(x) = 2x^5 - 3x^3 + 7x^2 - 9x$

polynomial, Degree of 5

b) $f(x) = 5x^3 + 12x^2 + \sqrt{9x}$

exponents must be positive integers

not a polynomial

$\sqrt{9x} = 9x^{\frac{1}{2}}$

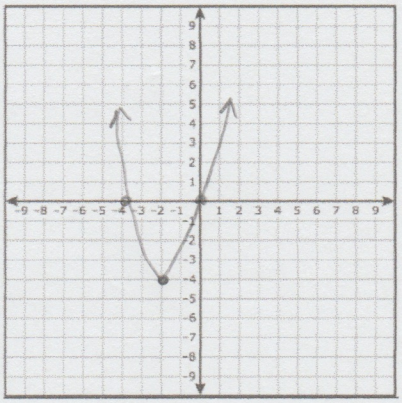
2.) (5 pts) Graph the quadratic function, which is given in standard form

$f(x) = (x + 2)^2 - 4$

vertex: $(-2, -4)$

$(x-h)^2 + k$

vertex: (h, k)



$(x+2)^2 - 4 = 0$

$+4 \quad +4$

$\sqrt{(x+2)^2} = \sqrt{4}$

$x + 2 = \pm 2$

$-2 \quad -2$

$x = 2 - 2$ and $x = -2 - 2$

$0 \qquad -4$

3.) (10 pts) Rewrite the quadratic function in standard form by completing the square. Then graph.

$f(x) = 2x^2 + 8x + 5 = 0$

y-intercept

$-5 \quad -5$

$2x^2 + 8x = -5$

$2(x^2 + 4x) = -5$

$\left(\frac{4}{2}\right)^2$

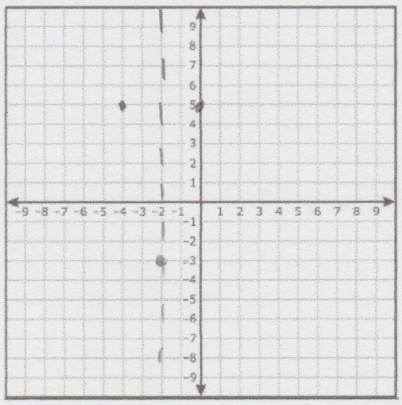
$2(x^2 + 4x + 4) = -5 + 8$

$2(x+2)^2 = 3$

$-3 \quad -3$

$2(x+2)^2 - 3$

vertex: $(-2, -3)$



Use reflection of y-intercept for the 2nd point — the x-intercepts are not great;

4.) (5 pts) Find all of the real zeros (and their state of multiplicities) for the polynomial.

$f(x) = 6x^2(x - 2)^4(x + 7)^3$

zeros: 0, 2, −7
multiplicity: 2, 4, 3

$\frac{6x^2}{6} = \frac{0}{6}$ $\quad x - 2 = 0$ $\quad x + 7 = 0$

$\sqrt{x^2} = \sqrt{0}$ $\quad +2\ +2$ $\quad -7\ -7$

$x = 0$ $\quad x = 2$ $\quad x = -7$

5.) (10 pts) Find a polynomial of minimum degree that has the given zeros.

-2, 0, 1, 3

$x = -2$ $\quad x = 0$ $\quad x = 1$ $\quad x = 3$

$+2\ +2$ $\quad -1\ -1$ $\quad -3\ -3$

$x + 2 = 0$ $\quad x = 0$ $\quad x - 1 = 0$ $\quad x - 3 = 0$

$(x+2)(x)(x-1)(x-3) = 0$

$(x+2)(x-1)$

$x^2 - x + 2x - 2$

$(x^2 + x - 2)(x - 3)$

$x^3 + x^2 - 2x - 3x^2 - 3x + 6$

$(x^3 - 2x^2 - 5x + 6)(x)$

$x^4 - 2x^3 - 5x^2 + 6x$

6.) (10 pts) For the polynomial function: (a) list each real zero and its multiplicity; (b) determine whether the graph touches or crosses at each x-intercept; (c) find the y-intercept; (d) sketch-ish the graph.

$f(x) = x^5 - 4x^3$

y-int: $(0)^5 - 4(0)^3 = 0$

$x^3(x^2 - 4)$

$x^3(x - 2)(x + 2)$

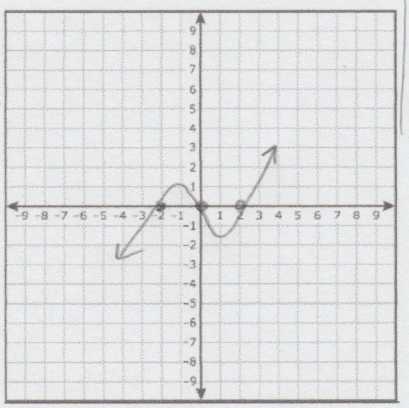
$\sqrt[3]{x^3} = \sqrt[3]{0}$ $\quad x - 2 = 0$ $\quad x + 2 = 0$

$+2\ +2$ $\quad -2\ -2$

a)

	$x = 0$	$x = 2$	$x = -2$
mult	3	1	1

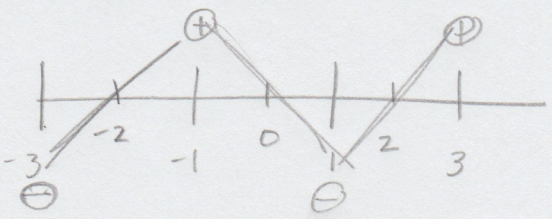
b) crosses @ −2, 0, 2



$(x^3)(x - 2)(x + 2)$

using −3, −1, 1, 3 based on zeros

−3	(−)	(−)	(−)	(−)
−1	(−)	(−)	(+)	(+)
1	(+)	(−)	(+)	(−)
3	(+)	(+)	(+)	(+)



7.) (7.5 pts each, 15 pts total) Divide the polynomials by either long division or synthetic division.

a) $(x^4 - 2x^3 - 7x^2 + 8x + 12) \div (x + 2)$

$$x^3 - 4x^2 + x + 6$$

$$x+2 \;\overline{)\; x^4 - 2x^3 - 7x^2 + 8x + 12}$$

$$-x^4 + 2x^3$$

$$-4x^3 - 7x^2$$

$$+4x^3 + 8x^2$$

$$x^2 + 8x$$

$$-x^2 + 2x$$

$$6x + 12$$

$$-6x + 12$$

$$0$$

$$\boxed{x^3 - 4x^2 + x + 6}$$

$$-2 \;\big|\; 1 \;\; -2 \;\; -7 \;\; 8 \;\; 12$$

$$\qquad\;\; -2 \;\; 8 \;\; -2 \;\; -12$$

$$\qquad 1 \;\; -4 \;\; 1 \;\; 6 \;\; 0$$

b) $(x^5 + 4x^4 + 3x^2 + 19x + 28) \div (x + 4)$

$$x^4 + 0x^3 + 0x^2 + 3x + 7$$

$$x+4 \;\overline{)\; x^5 + 4x^4 + 0x^3 + 3x^2 + 19x + 28}$$

$$-x^5 + 4x^4$$

$$0x^4 + 0x^3$$

$$-0x^4 - 0x^3$$

$$+0x^3 + 3x^2$$

$$0x^3 + 0x^2$$

$$3x^2 + 19x$$

$$-3x^2 + 12x$$

$$7x + 28$$

$$-7x + 28$$

$$0$$

$$\boxed{x^4 + 3x + 7}$$

$$-4 \;\big|\; 1 \;\; 4 \;\; 0 \;\; 3 \;\; 19 \;\; 28$$

$$\qquad\;\; -4 \;\; 0 \;\; 0 \;\; -12 \;\; -28$$

$$\qquad 1 \;\; 0 \;\; 0 \;\; 3 \;\; 7 \;\; 0$$

8.) (10 pts) For the function:

$$x^4 + 8x^3 + 9x^2 - 38x - 40$$

a) Find all potential zeros.

Factors of -40 and 1

Leading: ± 1

Ending: $\pm 40, \pm 1$
$\pm 20, \pm 2$
$\pm 10, \pm 4$
$\pm 8, \pm 5$

b) Find the number of possible *positive* zeros.

$f(x) = +x^4 + 8x^3 + 9x^2 - 38x - 40$ (1)

1

c) Find the number of possible *negative* zeros. 3, 1

$f(-x) = (-x)^4 + 8(-x)^3 + 9(-x)^2 - 38(-x) - 40$

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1 2 3

d) Attempt to find **3 zeros** using long division or synthetic division. Show all work.

various answers

-5 | 1 8 9 −38 −40
−5 −15 +30 40
1 3 −6 −8 0

-1 | 1 8 9 −38 −40
−1 −7 −2 40
1 7 2 −40 0

-4 | 1 8 9 −38 −40
−4 −16 28 +40
1 4 −7 −10 0

2 | 1 8 9 −38 −40
2 20 58 40
1 10 29 +20 0

9.) (10 pts) Find a polynomial of minimum degree with the following zeros:

$-4, 3 - i, 3 + i$

$x = -4$ $\quad x = 3 - i$ $\quad x = 3 + i$

$+4 \quad +4 \quad -3+i \quad -3+i \quad -3-i \quad -3-i$

$x + 4 = 0 \quad x - 3 + i = 0 \quad x - 3 - i = 0$

$i = \sqrt{-1}$

$(x - 3 + \sqrt{-1})(x - 3 - \sqrt{-1})$

$x^2 - 3x - x\sqrt{-1} - 3x + 9 + 3\sqrt{-1} + x\sqrt{-1} - 3\sqrt{-1} - (-1)$ (+1)

$x^2 - 6x + 10$

$(x^2 - 6x + 10)(x + 4)$

$x^3 - 6x^2 + 10x + 4x^2 - 24x + 40$

$\boxed{x^3 - 2x^2 - 14x + 40}$

10.) (10 pts) Given a zero of the polynomial, determine all other zeros (real or complex) and write the polynomial as a product of linear factors.

$x^4 + x^3 - 8x^2 + 4x - 48$, zero = $2i$

Since $2i$ is a zero, $-2i$ is also a zero.

$x = 2i \quad x = -2i$

$-2i \quad -2i \quad +2i \quad +2i$

$i = \sqrt{-1}$

$(x - 2\sqrt{-1})(x + 2\sqrt{-1})$

$x^2 + 2x\sqrt{-1} - 2x\sqrt{-1} - 4(-1)$

$x^2 + 4$

now we divide that through

$$\begin{array}{r|l} & x^2 + x - 12 \\ \hline x^2 + 0x + 4 & x^4 + x^3 - 8x^2 + 4x - 48 \\ & -x^4 - 0x^3 + 4x^2 \\ \hline & x^3 - 12x^2 + 4x \\ & -x^3 - 0x^2 - 4x \\ \hline & -12x^2 + 0x - 48 \\ & +12x^2 + 0x + 48 \\ \hline & 0 \end{array}$$

$x^2 + x - 12$

factors into $(x + 4)(x - 3)$

$(x+4)(x-3)(x - 2\sqrt{-1})(x + 2\sqrt{-1})$

$(x^2 + 4)$

11.) (5 pts each, 10 pts total) Find the domain and asymptotes (vertical and horizontal) of each of the following rational functions.

a) $\frac{x^2-4}{3x^2-8x+4}$

Domain: $3x^2-8x+4 \neq 0$

$(-\infty, \frac{2}{3}) \cup (\frac{2}{3}, 2) \cup (2, \infty)$ $\quad a=3 \quad b=-8 \quad c=4$

$x^2-4 = (x-2)(x+2)$

$-b \pm \sqrt{b^2-4ac} = 8 \pm \sqrt{(-8)^2-4(3)(4)}$

$2a \qquad\qquad 2(3)$

$3x^2-8x+4 = 3(x-2)(x-\frac{2}{3})$

$8 \pm \sqrt{64-48}$

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$\frac{(x-2)(x+2)}{3(x-2)(x-\frac{2}{3})}$

$\frac{8 \pm \sqrt{16}}{6} = \frac{8+4}{6} \quad \frac{8-4}{6}$

$x-2=0$

$+2 \quad +2$

$x=2$ is a hole

not an asymptote

$x \neq 2$ and $\frac{2}{3}$

vertical asymptote: $x=\frac{2}{3}$

$\lim_{x\to\infty} \frac{x^2-4}{3x^2-8x+4} = \frac{x^2}{3x^2} = \frac{1}{3}$ horizontal asymptote: $\frac{1}{3}$

b) $\frac{4x^2-3x+6}{8x^3-16x^2+8x}$

Domain: $8x^3-16x^2+8x \neq 0$

Domain:

$8x(x^2-2x+1) \neq 0$

$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

$8x(x-1)(x-1) \neq 0$

vertical asymptotes: 0, 1

$x \neq 0 \qquad x-1 \neq 0$

$+1 \quad +1$

$x \neq 1$

horizontal asymptotes: 0

$\frac{4x^2-3x+6}{8x^3-16x^2+8x}$

$\lim_{x\to\infty} \frac{4x^2}{8x^3} = \frac{1}{2x} = \frac{1}{\infty} = 0$

$4x^2-3x+6$

$a=4 \quad b=-3 \quad c=6$

$-b \pm \sqrt{b^2-4ac}$

$2a$

$3 \pm \sqrt{(-3)^2-4(4)(6)}$

$2(4)$

$3 \pm \sqrt{9-96}$

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imaginary no, zeros

Therefore, only asymptotes and no holes