

Key

Pre-Calculus Chapter 0 Pre-Test

Each question is worth 4 points each. Please show all work. Partial credit will be awarded for partially credit responses.

Solve for the indicated variable.

1.) $8x + 14 = 5x - 22$

$$\begin{array}{r} -5x \\ \hline 3x + 14 = -22 \end{array}$$

$$\begin{array}{r} -14 \\ \hline 3x = -36 \end{array}$$

$$\frac{3x}{3} = \frac{-36}{3}$$

$$x = -12$$

2.) $6(2y - 1) = 3(5y + 2)$

$$\begin{array}{r} 12y - 6 = 15y + 6 \\ -12y \quad -12y \end{array}$$

$$\begin{array}{r} -6 = 3y + 6 \\ -6 \quad -6 \end{array}$$

$$\frac{-12}{3} = \frac{3y}{3}$$

$$y = -4$$

3.) $\left(\frac{m}{4} = \frac{m}{20} + 16\right) 20$

$$\frac{20m}{4} = \frac{20m}{20} + 320$$

$$\frac{4m}{4} = \frac{320}{4}$$

$$\begin{array}{r} 5m = m + 320 \\ -m \quad -m \end{array}$$

$$m = 80$$

4.) $\left(\frac{5a}{6} - a = \frac{a}{4} - \frac{5}{2}\right) 12$

$$\frac{60a}{6} - 12a = \frac{12a}{4} - \frac{60}{2}$$

$$10a - 12a = 3a - 30$$

$$\begin{array}{r} -2a = 3a - 30 \\ -3a \quad -3a \end{array}$$

$$a = 6$$

$$\frac{-5a}{-5} = \frac{-30}{-5}$$

Solve by factoring.

$$5.) x^2 - 6x + 8 = 0$$

$$(x - 4)(x - 2) = 0$$

$$\begin{array}{r} x - 4 = 0 \\ +4 \quad +4 \\ \hline x = 4 \end{array}$$

$$\begin{array}{r} x - 2 = 0 \\ +2 \quad +2 \\ \hline x = 2 \end{array}$$

$$6.) 3x^2 - 2x - 16 = 0$$

$$(3x - 8)(x + 2) = 0$$

$$\begin{array}{r} 3x - 8 = 0 \\ +8 \quad +8 \\ \hline 3x = 8 \end{array}$$

$$\begin{array}{r} x + 2 = 0 \\ -2 \quad -2 \\ \hline x = -2 \end{array}$$

$$x = \frac{8}{3}$$

$$\begin{cases} x = \frac{8}{3} \\ x = -2 \end{cases}$$

$$7.) 9y^2 = 36y$$

$$-36y \quad -36y$$

$$9y^2 - 36y = 0$$

$$9y(y - 4) = 0$$

$$\begin{array}{r} 9y = 0 \\ \frac{9y}{9} \quad \frac{y - 4}{+4} \\ \hline y = 0 \quad y = 4 \end{array}$$

$$8.) 5a^2 = 25a - 20$$

$$-25a + 20 - 25a + 20$$

$$5a^2 - 25a + 20 = 0$$

$$5(a^2 - 5a + 4) = 0$$

$$5(a - 1)(a - 4) = 0$$

$$\begin{array}{r} a - 1 = 0 \\ +1 \quad +1 \\ \hline a = 1 \end{array}$$

$$\begin{array}{r} a - 4 = 0 \\ +4 \quad +4 \\ \hline a = 4 \end{array}$$

Solve using the square root method.

$$9.) \sqrt{(x - 4)^2} = \sqrt{64}$$

$$x - 4 = 8 \quad x - 4 = -8$$

$$+4 \quad +4 \quad +4 \quad +4$$

$$x = 12 \quad x = -4$$

$$x - 4 = \pm 8$$

Find vertex by completing the square. Graph.

$$10.) \quad x^2 - 4x = -10$$

$$+10 \quad +10$$

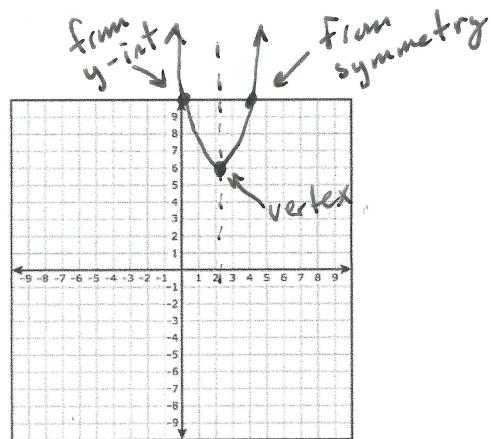
$$\left(\frac{-4}{2}\right)^2 = 4$$

$$(x^2 - 4x + 4) - 4 + 10 = 0$$

$$\cancel{x^2} \downarrow \quad \cancel{+4} \downarrow$$

$$(x - 2)^2 + 6$$

$$\boxed{\text{vertex: } (2, 6)}$$



Solve by using the quadratic formula.

$$11.) \quad 4x^2 - 2x + 11 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(11)}}{2(4)}$$

$$\frac{2 \pm \sqrt{4 - 176}}{8}$$

$$\frac{2 \pm \sqrt{-172}}{8}$$

$$\frac{2 \pm \sqrt{-4 \cdot 43}}{8}$$

$$\boxed{\frac{1 \pm i\sqrt{43}}{4}}$$

Specify any values that must be excluded from the solution set and then solve the rational equation.

$$12.) \frac{3}{w+4} + \frac{1}{w} = \frac{w^2}{w(w+4)}$$

$$\left(\frac{3}{w+4}\right)\left(\frac{w}{w}\right) + \frac{1}{w} \frac{(w+4)}{(w+4)} = \frac{w^2}{w(w+4)}$$

$$3w + w + 4 = w^2$$

$$\begin{array}{r} 4w + 4 = w^2 \\ -4w -4 \\ \hline -4w -4 \end{array}$$

Once the denominators are the same, they cancel.

$$w^2 - 4w - 4 = 0$$

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-4)}}{2(1)}$$

$$\frac{4 \pm \sqrt{16+16}}{2}$$

$$\frac{4 \pm \sqrt{32}}{2} \quad \frac{4 \pm 4\sqrt{2}}{2}$$

$$z = 2\sqrt{2}$$

Solve the radical equation for the given variable.

$$13.) \sqrt[3]{4(t+3)^{\frac{3}{2}}} = \frac{16}{4}$$

$$\left((t+3)^{\frac{2}{3}}\right)^{\frac{3}{2}} = (4)^{\frac{3}{2}}$$

$$t+3 = \pm 2^3$$

$$\begin{array}{r} t+3 = 8 \\ -3 -3 \\ \hline t = 5 \end{array}$$

$$t+3 = (-2)^3$$

$$t+3 = -8$$

$$\begin{array}{r} -3 -3 \\ \hline t = -11 \end{array}$$

$$14.) (\sqrt{2x+8})^2 = (x-8)^2$$

$$2x+8 = (x-8)(x-8)$$

$$\begin{array}{r} 2x+8 = x^2 - 16x + 64 \\ -2x -8 \quad -2x -8 \\ \hline 0 = x^2 - 18x + 56 \end{array}$$

$$(x-14)(x-4) = 0$$

$$\begin{array}{l} x-14=0 \quad x-4=0 \\ +14 +14 \\ \hline x=14 \quad x=4 \end{array}$$

Note: Since we have

$$\sqrt{2x+8}$$

For all solutions,

$$\begin{array}{r} 2x+8 \geq 0 \\ -8 -8 \\ \hline 2x \geq -8 \end{array}$$

$$\begin{array}{r} \frac{2x}{2} \geq \frac{-8}{2} \\ x \geq -4 \end{array}$$

Solve either through u substitution or factoring.

$$15.) \quad 3t^{2/3} - t^{1/3} - 2 = 0$$

$$u = t^{1/3}$$

$$3u^2 - u - 2 = 0$$

$$\left(t^{\frac{1}{3}}\right)^3 = \left(-\frac{2}{3}\right)^3 \quad \left(t^{\frac{1}{3}}\right)^3 = 1^3$$

$$(3u+2)(u-1) = 0$$

$$-\frac{2}{3} \cdot -\frac{2}{3} \cdot \frac{-2}{3}$$

$$\begin{array}{l} 3u+2=0 \\ -2-2 \\ u-1=0 \\ +1+1 \\ u=-\frac{2}{3} \end{array}$$

$$\begin{array}{l} 3u=-2 \\ \frac{3u}{3}=\frac{-2}{3} \\ u=-\frac{2}{3} \end{array}$$

$$\boxed{t=1}$$

$$\boxed{\frac{-8}{27} = t}$$

$$16.) \quad (x-2)^4 - 8(x-2)^2 + 16 = 0$$

$$u^2 - 8u + 16 = 0$$

$$u = (x-2)^2$$

$$(u-4)(u-4) = 0$$

$$\sqrt{(x-2)^2} = \sqrt{4}$$

$$\begin{array}{l} u-4=0 \\ +4+4 \\ u=4 \end{array}$$

$$\begin{array}{l} x-2=2 \\ +2+2 \\ x=4 \end{array}$$

$$x-2 = \pm 2$$

$$\begin{array}{l} x-2=-2 \\ +2+2 \\ x=0 \end{array}$$

Solve the absolute value equation.

$$17.) \quad |2x - 3| = 11$$

$$\begin{array}{ll} 2x-3=11 & 2x-3=-11 \\ +3+3 & +3+3 \\ 2x=14 & 2x=-8 \\ \frac{2x}{2}=\frac{14}{2} & \frac{2x}{2}=\frac{-8}{2} \\ x=7 & x=-4 \end{array}$$

$$\boxed{x=-4}$$

18.) $2|3x - 6| - 12 = -6$

$$\frac{2|3x - 6|}{2} = \frac{6}{2}$$

$$|3x - 6| = 3$$

$$3x - 6 = 3 \quad 3x - 6 = -3$$

$$+6 \quad +6 \quad +6 \quad +6$$

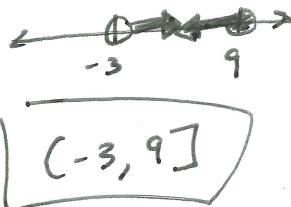
$$\begin{array}{r} 3x = 9 \\ \hline 3 \end{array} \quad \begin{array}{r} 3x = 3 \\ \hline 3 \end{array}$$

$$x = 3 \quad x = 1$$

Rewrite in interval notation and graph.

19.) $-3 < x \leq 9$

$$-3 < x \quad x \leq 9$$



Solve each linear inequality and express the solution set in interval notation.

20.) $-11 \leq -2x + 3 < -3$

$$\frac{-14 \leq -2x}{-2} < \frac{-6}{-2}$$

$$7 \geq x > 3$$

$$(-3, 7]$$



$$(-\infty, -5] \cup [7, \infty)$$

21.) $x^2 - 2x - 35 \geq 0$

POSITIVE

$$(x - 7)(x + 5) \geq 0$$

$$x - 7 = 0 \quad +7 +7$$

$$x + 5 = 0 \quad -5 -5$$

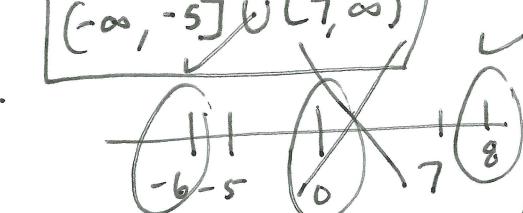
$$x = 7$$

$$-6: (-6 - 7)(-6 + 5) \geq 0$$

$$(-)(-) \geq 0 \quad (+) \geq 0 \quad \text{true}$$

$$0: (0 - 7)(0 + 5) \geq 0$$

$$(-)(+) \geq 0 \quad (-) \geq 0 \quad \text{false}$$



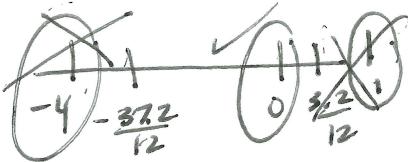
$$8: (8 - 7)(8 + 5) \geq 0$$

$$(+)(+) \geq 0$$

true

6

Not pretty...



$$22.) \quad -17x + 5 > 6x^2 \\ +17x - 5 \quad +17x - 5$$

$$\frac{-17 \pm \sqrt{(17)^2 - 4(6)(-5)}}{2(6)}$$

$$\left[\frac{-17 - \sqrt{409}}{12}, \frac{-17 + \sqrt{409}}{12} \right]$$

NEGATIVE

$$0 > 6x^2 + 17x - 5$$

$$\frac{-17 + 20.2}{12}$$

$$\frac{-17 - 20.2}{12}$$

$$\frac{3.2}{12}$$

$$\frac{-37.2}{12}$$

$$\frac{-17 \pm \sqrt{289 + 120}}{12}$$

$$\frac{-17 + \sqrt{409}}{12}$$

$$\frac{-17 \pm \sqrt{409}}{12}$$

$$\frac{-17 - \sqrt{409}}{12}$$

$$0 > 6(-4)^2 + 17(-4) - 5$$

$$0 > 6(16) - 68 - 5$$

$$0 > 96 - 68 - 5$$

$$0 > 23$$

false

$$0 > 6(0)^2 + 17(0) - 5$$

$$0 > -5$$

true

$$0 > 6(1)^2 + 17(1) - 5$$

$$0 > 6 + 17 - 5$$

$$0 > +18 \quad \text{false}$$

Solve each rational inequality and express the solution set in interval notation.

$$23.) \quad \frac{x^2 - 36}{x+6} \geq 0$$

$$x-6=0$$

$$x+6 \neq 0$$

POSITIVE

$$\frac{(x-6)(x+6)}{(x+6)} \geq 0$$

$$\frac{(-)(-)}{(-)} \geq 0$$

\leftrightarrow ≥ 0

false

$$\frac{(0-6)(0+6)}{(0+6)} \geq 0$$

$$\frac{(-)(+)}{(+)} \geq 0$$

$\rightarrow 0$ false!

$$[6, \infty)$$

$$24.) \quad \frac{\frac{2t^2}{t-3} \geq -t}{t+t-tt}$$

POSITIVE

$$\frac{2t^2}{t-3} + t \geq 0$$

$$\frac{2t^2 + t^2 - 3t}{t-3} \geq 0$$

$$\frac{t-1}{t+1} = 0 \quad t=1$$

$$\frac{t-3}{t+3} = 0 \quad t=3$$

$$\frac{2t^2}{t-3} + \frac{t(t-3)}{t-3} \geq 0$$

$$\frac{3t^2 - 3t}{t-3} \geq 0$$

$$t=0 \quad t=3$$

$$\frac{34}{(4-1)} \quad (4-1)$$

$$[0, 1] \cup (3, \infty)$$

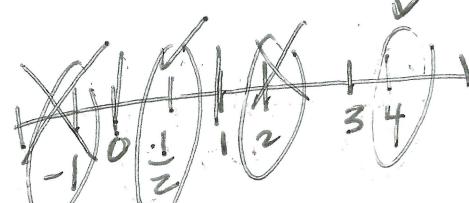
$$\frac{3t(t-1)}{t-3} \geq 0$$

$$\frac{3(-1)(-1-1)}{-1-3} \geq 0$$

$$\frac{(-)(-)}{(-)} \geq 0 \quad \text{false}$$

$$\frac{(+)(+)}{(+)} \geq 0$$

$\rightarrow 0$ true!



$$\frac{3(2)(2-1)}{2-3} \geq 0$$

$$\frac{(+)(+)}{(-)} \geq 0$$

$\rightarrow 0$ false

$$\frac{3(1)(1-1)}{(1-3)} \geq 0$$

$$\frac{(+)(-)}{(-)} \geq 0$$

$\rightarrow 0$ true

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Solve the absolute value inequality and express the solution set in interval notation.

25.) $|x + 2| < 5$

$$x + 2 < 5$$

$$-2 \quad -2$$

$$x < 3$$

$$x + 2 > -5$$

$$-2 \quad -2$$

$$x > -7$$

$$(-7, 3)$$

