

Nate was hit by a semi-truck and sued them for \$75,000

Nate wants to use his money to buy \$1,000,000 worth of rubber ducks. If he is able to get an 8% interest rate compounded continuously, how many years will it take?

$P$  = initial amount  
 $r$  = rate

$$A = P e^{rt}$$
$$\frac{\$1,000,000}{\$75,000} = \frac{\$75,000 e^{0.08t}}{\$75,000}$$

$$\ln\left(\frac{1,000,000}{75,000}\right) = \ln\left(e^{0.08t}\right)$$

$$\frac{\ln\left(\frac{1,000,000}{75,000}\right)}{0.08} = \frac{0.08t}{0.08}$$

$$t = \frac{\ln\left(\frac{1,000,000}{75,000}\right)}{0.08} = \frac{\ln(13.\bar{3})}{0.08}$$
$$= \boxed{32.4 \text{ years}}$$

$$\log(2^{3x}) = 28$$

Logs are exponents

$$\log(2^{3x}) = \log 28$$

$$x = \frac{\log(28)}{3 \log(2)}$$

$$\frac{3x \log(2)}{3 \log(2)} = \frac{\log 28}{3 \log(2)}$$

$$\log(8^{2x-3}) = 82$$

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$$\frac{(2x-3)(\log(8))}{\log(8)} = \frac{\log(82)}{\log(8)}$$

$$2x - 3 = \frac{\log(82)}{\log(8)}$$

$$x = \frac{\frac{\log(82)}{\log(8)} + 3}{2} = 2.54$$

$$\frac{2x}{2} = \frac{\frac{\log(82)}{\log(8)} + 3}{2}$$

$$\log_x b = t \quad \begin{array}{l} \leftarrow \text{exponent} \\ \leftarrow \text{base} \end{array} \quad \longrightarrow \quad x^t = b$$

Logarithmic form exponential form

$$\log_2 x = 5 \quad \begin{array}{l} 2^5 = x \\ \boxed{32 = x} \end{array}$$

$$\log_9 \boxed{512} = x$$

$$\log(9^x) = \log(512)$$

$$x \log(9) = \log(512)$$

$$\frac{x \log(9)}{\log(9)} = \frac{\log(512)}{\log(9)}$$

$$x = \frac{\log 512}{\log 9} = \boxed{2.84}$$

$$\log_{16} 1082 = X$$

$$\log (16^X) = \log (1082)$$

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$$\frac{X \log (16)}{\log (16)} = \frac{\log (1082)}{\log (16)}$$

$$X = \frac{\log (1082)}{\log (16)} = \boxed{2.51}$$

Expanding Logarithms

$$\log \left( \frac{(10,000)(1,000)}{100} \right) = \log \frac{10,000,000}{100}$$

$$\log \left( \frac{10,000 * 1,000}{100} \right) = 5$$

$$\log_{10} (100,000) = X$$

$$10^X = 100,000$$

$$\left[ \log \frac{(10^4) * (10^3)}{10^2} \right]$$

$$10^4 * 10^3 = 10^{4+3}$$

$$X = 5$$

$$\frac{10^7}{10^2} = 10^{7-2} = 10^5$$

$$\log \left( \frac{10^7}{10^2} \right) \quad \log 10^5 = 5$$

# Expand Logs

$$\log \frac{10^4 * 10^3}{10^2} = \log 10^4 + \log 10^3 - \log 10^2$$

$$\log_{10} 10^4 = X$$

$10^X = 10^4$

$$4 + 3 - 2$$
$$7 - 2 = 5$$

$$\log \frac{ac}{de} = \log a + \log c - \log d - \log e$$

$$\log \frac{a^7 b^8}{c^4} = 7 \log a + 8 \log b - 4 \log c$$

$$\log_2 \frac{x^9}{y^7 z^3} = 9 \log_2 x - 7 \log_2 y - 3 \log_2 z$$

Contract — write as a single log.

$$\curvearrowleft 3 \log_4 X + 2 \log_4 Y - 8 \log_4 Z$$

$$\boxed{\log_4 \frac{X^3 Y^2}{Z^8}}$$

$$\log_b \frac{X^{\frac{1}{2}}}{Y^{\frac{2}{3}}} = \log_b X^{\frac{1}{2}} - \log_b Y^{\frac{2}{3}}$$

$$\boxed{\frac{1}{2} \log_b X - \frac{2}{3} \log_b Y}$$

$$\log_8 (\sqrt{r} \sqrt[3]{t}) = \log_8 \sqrt{r} + \log_8 \sqrt[3]{t}$$
$$\log_8 r^{\frac{1}{2}} + \log_8 t^{\frac{1}{3}}$$

$$\boxed{\frac{1}{2} \log_8 r + \frac{1}{3} \log_8 t}$$

$$\log (X^2) (\sqrt{X+5}) \quad \log X^2 + \log \sqrt{X+5}$$

$$2 \log X + \frac{1}{2} \log (X+5)$$

$$\log_8 \frac{X^2 + 3X - 18}{X^2 - 4X + 3} = \log_8 \frac{(X+6)(X-3)}{(X-3)(X-1)}$$

$$\log_8 (X+6) + \log_8 (X-3) - \log_8 (X-3) - \log_8 (X-1)$$

$$\log_8 (X+6) - \log_8 (X-1)$$

$$\log_8 \frac{X+6}{X-1} = 2$$

$$8^2 = \frac{X+6}{X-1} (X-1) (64) = \left( \frac{X+6}{X-1} \right) (X-1)$$

$$X \neq 1$$

$$64(X-1) = X+6$$

$$64X - 64 = X + 6$$

$$\begin{array}{r} -X \qquad -X \\ 63X - 64 = 6 \\ +64 \quad +64 \end{array}$$

$$\frac{63X}{63} = \frac{70}{63}$$

$$X = \frac{70}{63} \div 7 = \frac{10}{9}$$

