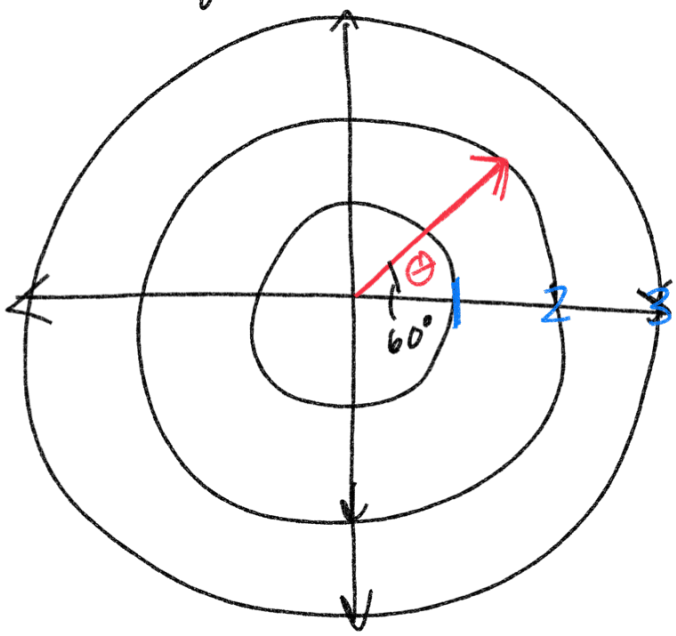
MTH-PT Trigonometry Session 25 4/27

Vector → magnitude and direction

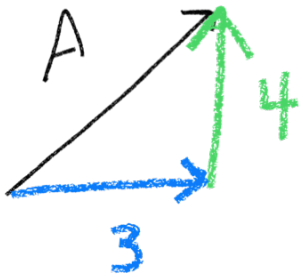


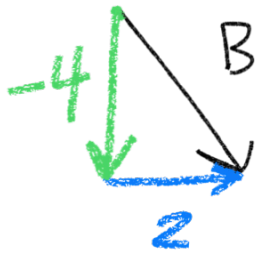
(r, θ)

$(2, \theta)$

$(2, 60°)$

$(2, \frac{\pi}{3})$





A: <3, 4>

B: <2, -4>

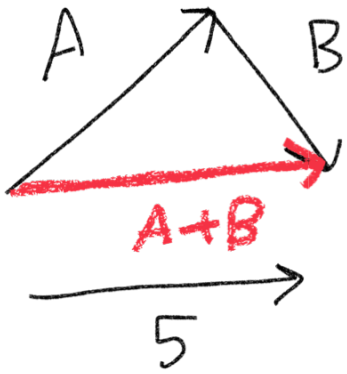
A + B

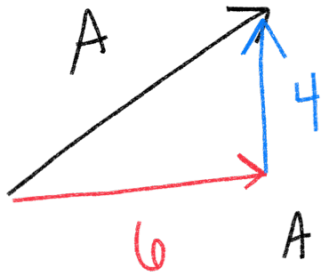
$(A_x + B_x)\ (A_y + B_y)$

$(3 + 2)\ (4 + (-4))$

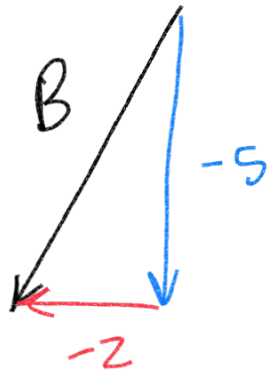
$\langle 5, 0 \rangle$

A + B





A: (6, 4)



B (-2, -5)

$(A+B)$

$\underbrace{6+(-2)}_{x} \quad \underbrace{4+(-5)}_{y}$

$\boxed{(4, -1)}$

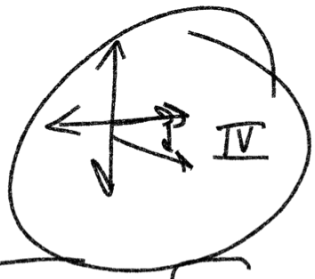
$(A-B)$ A: (6, 4) B (-2, -5)

$-B\ (2, 5)$

$\underbrace{6+2}_{8} \quad \underbrace{4+5}_{9}$

$\boxed{(8, 9) \text{ or } \langle 8, 9\rangle}$

$A+B \quad (4, -1) \quad (r, \theta)$

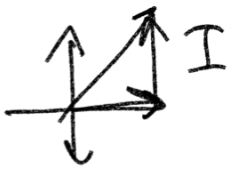


$r = \sqrt{x^2+y^2} = \sqrt{4^2+(-1)^2} = \sqrt{16+1} = \sqrt{17}$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-1}{4} = -14^\circ \quad (\sqrt{17}, -14^\circ)$

A - B $\langle 8, 9 \rangle \longrightarrow (r, \theta)$

$r = \sqrt{x^2 + y^2} = \sqrt{8^2 + 9^2} = \sqrt{64 + 81}$



$\sqrt{145}$

$\theta = \tan^{-1} \frac{y}{x}$

$\tan^{-1} \frac{9}{8} = 48.4^\circ$

$\boxed{(\sqrt{145}, 48.4)}$

$\vec{u} \cdot \vec{v}$ dot product

$\vec{u} : \langle -4, 8 \rangle$

$\vec{v} : \langle -2, -7 \rangle$

$$(\vec{u}_x)(\vec{v}_x) + (\vec{u}_y)(\vec{v}_y)$$

$$(-4)(-2) + (8)(-7)$$

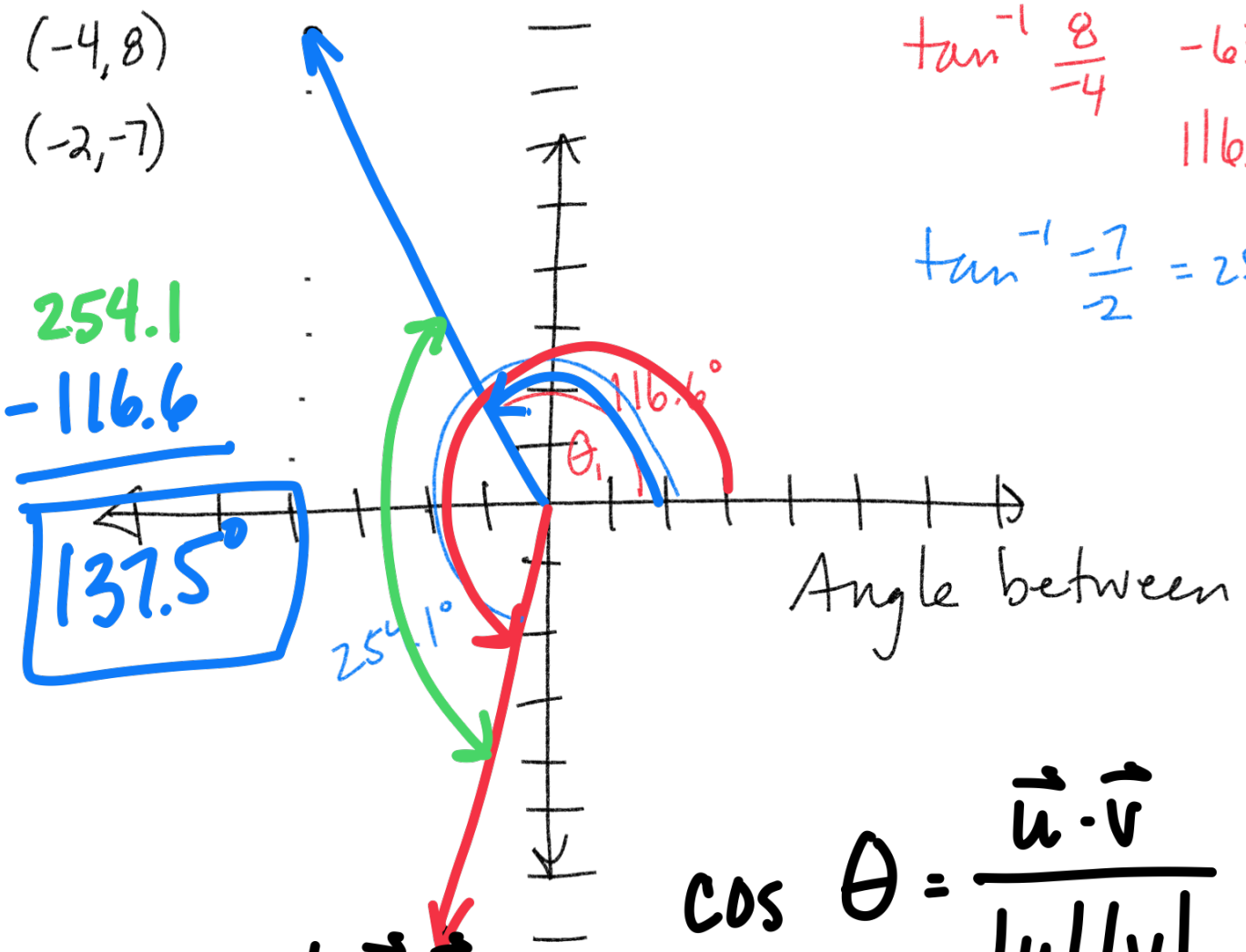
$$8 + (-56) = \boxed{-48}$$

Angle between two vectors

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|u||v|}$$

$|u|$ = magnitude of u

$|v|$ = magnitude of v



$\tan^{-1}\frac{8}{-4}$ $-63.4°$

116.6

$\tan^{-1}\frac{-7}{-2} = 254.1°$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|u||v|}$$

$$\theta = \cos^{-1}\frac{\vec{u}\cdot\vec{v}}{|u||v|}$$

$\vec{u}\cdot\vec{v} = -48$

$\frac{-48}{(\sqrt{80})(\sqrt{58})}$

$u = (-4, 8)$

$|u| = \sqrt{u_x^2 + u_y^2} = \sqrt{-4^2 + 8^2} = \sqrt{16+64} = \sqrt{80}$

$v = (-2, -7)$

$|v| = \sqrt{-2^2 + -7^2} = \sqrt{53}$

$$\theta = \cos^{-1}\frac{-48}{(\sqrt{80})(\sqrt{58})} = \boxed{137.5°}$$