Sum/Difference

b) $\sin x \cos(8x) - \cos x \sin(8x)$

$\sin(A-B)$ $\sin A \cos B - \cos A \sin B$

$\sin A-B$

$\sin(x-8x) = \boxed{\sin(-7x)}$

or

Even/Odd $\sin(-\theta) = -\sin\theta$ $\boxed{-\sin(7x)}$

4.) (10 pts total, 5 pts each) Use double angle identities to solve each of the following.

Double Angles

a) $If\ \cos x = \frac{5}{24}\ and\ \sin x < 0,\ find\ \tan(2x)$

(b) Double angle

$\frac{\sin 2x}{2} = \frac{2\boxed{\sin x \cos x}}{2}$

$x = 15$

$\left[\frac{\sin 2x}{2}\right] = \sin x \cos x$

$\frac{\sin 2(15)}{2} = \sin 15 \cos 15$

b) $\boxed{\sin 15° \cos 15°}$

$\frac{\sin 30}{2} = \frac{\frac{1}{2}}{2}$ $\frac{1}{2} \div 2$

$\frac{1}{2} * \frac{1}{2} = \boxed{\frac{1}{4}}$

Option 2

Product to Sum

$\sin\theta \cos\phi = \frac{[\sin(\theta+\phi) + \sin(\theta-\phi)]}{2}$

$\theta = 15$

$\phi = 15$

$\frac{\sin(15+15) + \sin(15-15)}{2}$

Option #3

$\sin 15 \cos 15$

$\sin(45-30) \cos(45-30)$

$\frac{\sin 30 + \sin 0}{2}$

$\frac{\frac{1}{2} + 0}{2} = \boxed{\frac{1}{4}}$

Half Angle

5.) (10 pts total, 2 pts each) Find the exact values of each problem

a) $\cos 15°$

$\sqrt{\cos^2\theta} = \sqrt{\frac{1+\cos 2\theta}{2}}$

$\theta = 15$

$\cos\theta = \pm\sqrt{\frac{1+\cos 2\theta}{2}}$

$2\theta = 2(15) = 30$

$\cos 15 = \pm\sqrt{\frac{1+\cos 30}{2}}$

$\boxed{+\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}}$

$= \pm\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$

$\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}}$ $\boxed{\sqrt{\frac{2+\sqrt{3}}{4}}}$

b) $\tan 202.5°$

$\tan \frac{405}{2}$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

$\tan 202.5 = \pm\sqrt{\frac{1-\cos 405}{1+\cos 405}}$

$405°$
-360
45

$\pm\sqrt{\frac{1-\cos 45}{1+\cos 45}}$

6.) (5 pts total) Write the product as a sum or difference.

Product to Sum

$4\sin(3x)\sin(4x)$

$\theta = 3x$

$\phi = 4x$

$4\sin\theta\sin\phi = \cancel{4}\left(\frac{\cos(\theta-\phi)-\cos(\theta+\phi)}{\cancel{2}}\right)$

$2(\cos(\theta-\phi)-\cos(\theta+\phi))$

$\cos(-x) = \cos x$

$2[\cos(3x-4x)-\cos(3x+4x))$

$\boxed{2[\underbrace{\cos(-x)}-\cos(7x)]}$

$\boxed{2[\cos(x)-\cos(7x)]}$

Sum-to-product

7.) (5 pts total) Write the expression as a product.

$\theta = 8x \quad \phi = 3x$

$cos\,(8x) \; + \; cos\,(3x)$

$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$

$2\cos\left(\frac{8x+3x}{2}\right)\cos\left(\frac{8x-3x}{2}\right)$

$\boxed{2\cos\left(\frac{11x}{2}\right)\cos\left(\frac{5x}{2}\right)}$

2.) (40 pts total, 5 pts each) Verify each of the following trigonometric identities.

a) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

b) $\tan x + \cot x = \csc x \sec x$

$$\frac{\sin x}{\sin x}\left(\frac{\sin x}{\cos x}\right) + \left(\frac{\cos x}{\sin x}\right)\frac{\cos x}{\cos x}$$

$$\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} = \frac{1}{\sin x} \cdot \frac{1}{\cos x}$$

$$\csc x \ \sec x$$

c) $\frac{2 - \sin^2 x}{\cos x} = \sec x + \cos x$

Duh!

$\sin^2 x + \cos^2 x = 1$

d) $\frac{1}{csc^2 x} + \frac{1}{sec^2 x} = 1$

$$\frac{1}{\frac{1}{\sin^2 x}} + \frac{1}{\frac{1}{\cos^2 x}} = 1$$

$1 \div \frac{1}{\sin^2 x}$ $\quad 1 \div \frac{1}{\cos^2 x}$

$1 * \sin^2 x$ $\quad 1 * \cos^2 x$

e) $\frac{1}{1 - sin\, x} + \frac{1}{1 + sin\, x} = 2sec^2 x$

f) $\frac{sin^2 x}{1 - cos\, x} = 1 + cos\, x$

$\sin^2 x + \cos^2 x = 1$

$-\cos^2 x \quad -\cos^2 x$

$\sin^2 x = 1 - \cos^2 x$

$$\frac{1 - \cos^2 x}{1 - \cos x}$$

Difference of Squares

$1 - \cos^2 x =$

$(1 + \cos x)(1 - \cos x)$

$$\frac{(1 + \cos x)\cancel{(1 - \cos x)}}{\cancel{1 - \cos x}} = 1 + \cos x$$