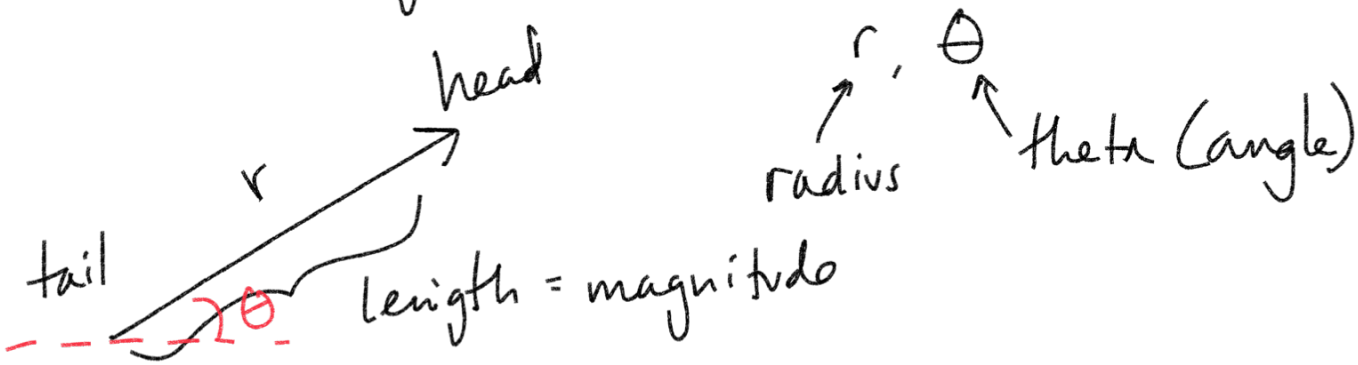
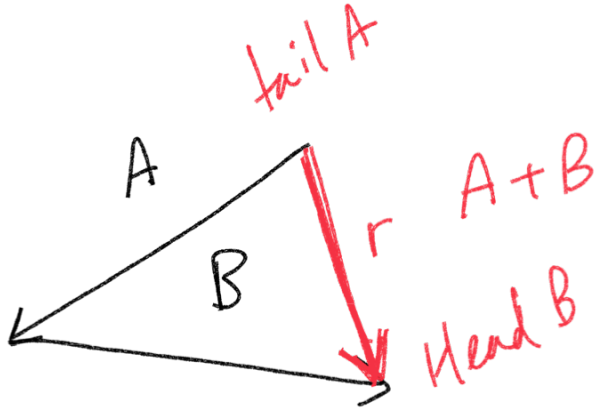
MTH-PT Trigonometry Session 24 4/24

vector → magnitude and a direction

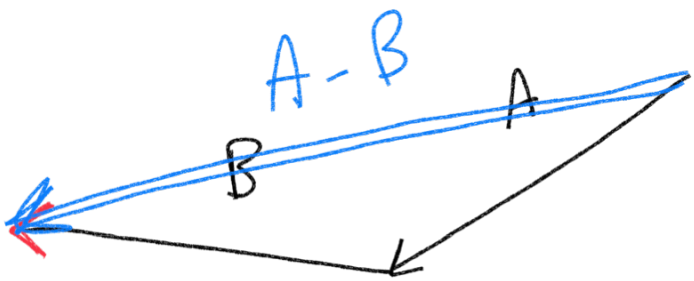


A + B

Add Head A to Tail B



A - B

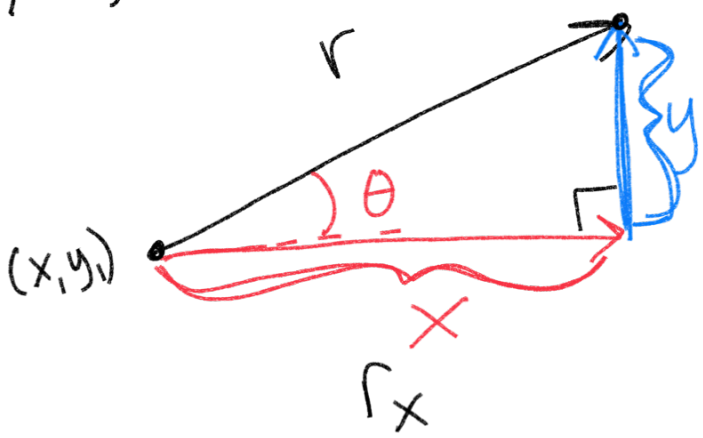


Vectors

X - components

Y - components

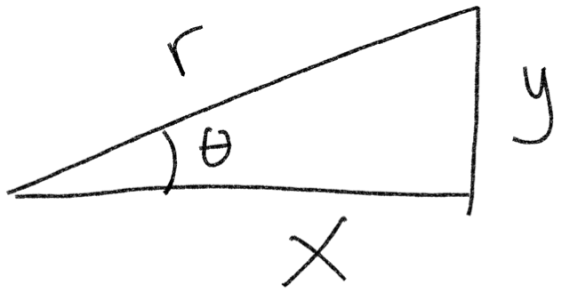
(r, θ)



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

r_y $r = \sqrt{x^2 + y^2}$

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$$



$$r(\cos\theta) = \left(\frac{x}{r}\right) r$$

$$\boxed{x = r\cos\theta}$$

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$$

$$r(\sin\theta) = \left(\frac{y}{r}\right) r$$

$$\boxed{y = r\sin\theta}$$

For funsies...

in unit circle,

radius = 1 $r = 1$

$$r = \sqrt{x^2 + y^2}$$

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$r = \sqrt{(r\cos\theta)^2 + (r\sin\theta)^2}$$

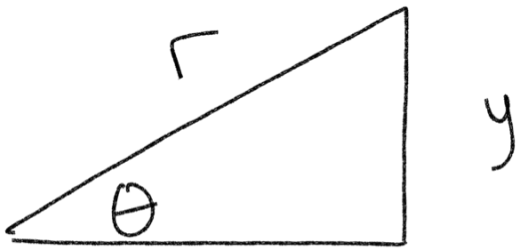
$$(1) = \left(\sqrt{\cos^2\theta + \sin^2\theta}\right)^2$$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

Vector (r, θ)

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$



$$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$$

$$\tan\theta = \frac{y}{x}$$

vector

$\vec{u} + \vec{V}$

$u: \langle 2, 6 \rangle$

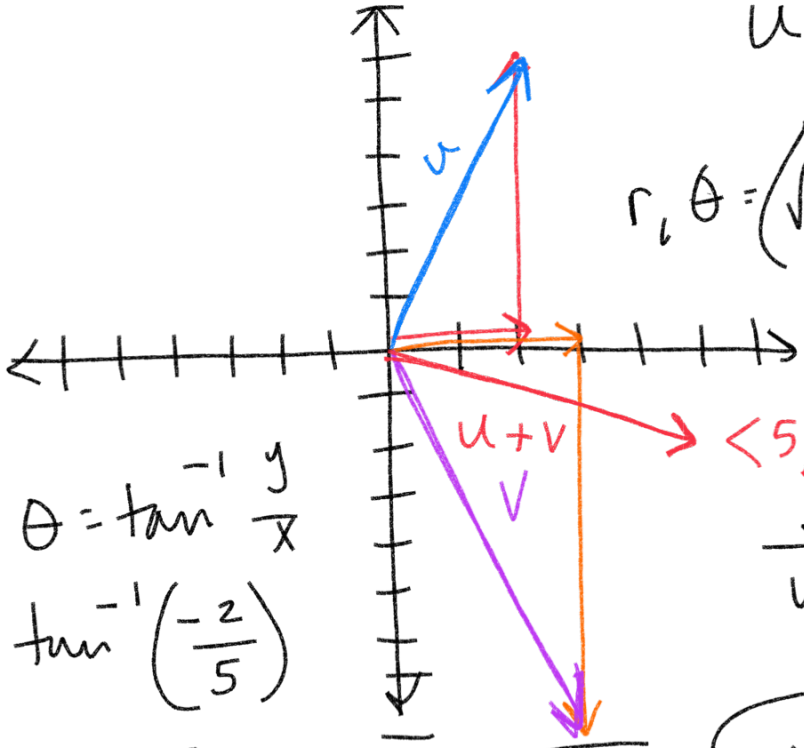
$V: \langle 3, -8 \rangle$

$\vec{u} + \vec{v}$

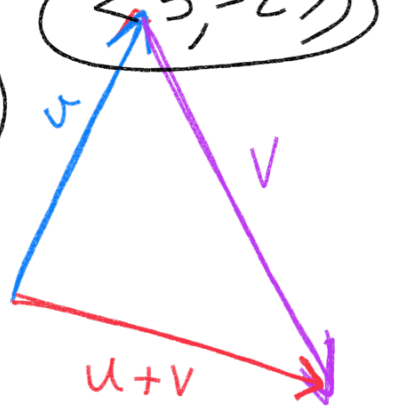
$$u \langle \overset{x}{2} \quad \overset{y}{6} \rangle$$

$$V + \langle 3 \quad -8 \rangle$$

$$\langle 5, -2 \rangle$$



$r, \theta = (\sqrt{29}, -21.8°)$



$$\theta = \tan^{-1}\frac{y}{x}$$

$$\tan^{-1}\left(\frac{-2}{5}\right)$$

-21.8

$$\vec{u} + \vec{v} = \langle u_x + V_x, u_y + V_y \rangle$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(5)^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29}$$

$-\vec{u} + \vec{v}$

$\vec{u} = \langle 2, 6 \rangle$

$\vec{v} = \langle \overset{x}{3}, \overset{y}{-8} \rangle$

Find components $\langle 1, -14 \rangle$ $-\vec{u} = \langle \overset{x}{-2}, \overset{y}{-6} \rangle$

Find (r, θ)

$$-\vec{u} + \vec{v} = \langle (-2 + 3), (-6 + -8) \rangle$$

$$\langle 1, -14 \rangle$$

$+x \quad -y$

$$r = \sqrt{x^2 + y}$$

$$\sqrt{1^2 + (-14)^2} = \sqrt{1 + 196} = \sqrt{197} = 14.0$$

$$\theta = \tan^{-1} \frac{-14}{1} = \tan^{-1} -14 = -85.9^\circ$$

$$(r, \theta) = (\sqrt{197}, -85.9^\circ) \text{ or } (14.0, -85.9^\circ)$$

Work = force • distance

Dot Product

$\vec{u} \cdot \vec{v}$

$\vec{u} : \langle 2, 6 \rangle$

$\vec{v} : \langle 3, -8 \rangle$

$$(u_x)(\vec{v}_x) + (\vec{v}_y)(\vec{u}_y)$$

$$(2)(3) + (-8)(6)$$

$$6 + -48 = \boxed{-42}$$

Angle Between two Vectors

$$\cos\theta = \frac{u \cdot v}{|u| * |v|}$$

$$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u| * |v|}\right) = \cos^{-1}\left(\frac{-42}{\sqrt{10} * \sqrt{73}}\right) = \cos^{-1}\left(\frac{-21}{\sqrt{730}}\right) = 141^\circ$$

$$|u| = \sqrt{(u_x)^2 + (u_y)^2} \qquad |v| = \sqrt{3^2 + (-8)^2} = \sqrt{9+64} = \sqrt{73}$$

$$\sqrt{2^2 + 6^2} = \sqrt{4+36} = \sqrt{40}$$

$$= \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$$