Calculus & Trigonometry

Chapter 6 Pre-Test

1.) (20 pts total, 5 pts each) Simplify each of the following trigonometric expressions.

a) $(\sin^2 x)(\cot^2 x + 1)$

1.) sin/cos

2.) $\sin^2 x + \cos^2 x = 1$

SHEET

$\sin^2 x \left(\frac{\cos^2 x}{\sin^2 x} + 1\right)$

$\frac{\cancel{\sin^2 x}\cos^2 x}{\cancel{\sin^2 x}} + \sin^2 x$

$\cos^2 x + \sin^2 x = \boxed{1}$

b) $(\sin x - \cos x)(\sin x + \cos x)$

Difference of squares

even

c) $\frac{1 - \cos^4 x}{1 + \cos^2 x} = \frac{(1 - \cos^2 x)\cancel{(1 + \cos^2 x)}}{\cancel{1 + \cos^2 x}}$

$1 - \cos^2 x = \boxed{\sin^2 x}$

$\sin^2 x + \cos^2 x = 1$

$-\cos^2 x \quad -\cos^2 x$

$\sin^2 x = 1 - \cos^2 x$

d) $1 - \frac{\sin^2 x}{1 - \cos x}$

2.) (40 pts total, 5 pts each) Verify each of the following trigonometric identities.

a) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

$(\sin x + \cos x)(\sin x + \cos x) + (\sin x - \cos x)(\sin x - \cos x)$

$\underbrace{\sin^2 x + \cancel{2\sin x \cos x} + \cos^2 x}_{1} + \underbrace{\sin^2 x - \cancel{2\sin x \cos x} + \cos^2 x}_{1}$

$1 + 1 = 2$

b) $\tan x + \cot x = \csc x \sec x$

c) $\dfrac{2 - \sin^2 x}{\cos x} = \sec x + \cos x$

Scenario #1 $\dfrac{2 - \sin^2 x}{\cos x} = \dfrac{1}{\cos x} + (\cos x)\left(\dfrac{\cos x}{\cos x}\right)$

$\sin^2 x + \cos^2 x = 1$

$-\sin^2 x \qquad -\sin^2 x$

$\cos^2 x = \boxed{1 - \sin^2 x}$

$\dfrac{1}{\cos x} + \dfrac{\cos^2 x}{\cos x}$

$\dfrac{1 + \cos^2 x}{\cos x}$

Scenario #2

$\dfrac{2 - \sin^2 x}{\cos x} = \dfrac{1 + 1 - \sin^2 x}{\cos x}$

$\dfrac{1 + \cos^2 x}{\cos x}$

$\dfrac{1}{\cos x} + \cos x = \sec x + \cos x$

$\dfrac{1 + 1 - \sin^2 x}{\cos x}$

$\boxed{\dfrac{2 - \sin^2 x}{\cos x}}$

d) $\frac{1}{csc^2 x} + \frac{1}{sec^2 x} = 1$

$\frac{6}{6} \cdot \frac{1}{4} + \frac{1}{6} \cdot \frac{4}{4} \quad \frac{\triangle}{\triangle} \cdot \frac{1}{\square} + \frac{1}{\triangle} \cdot \frac{\square}{\square}$

e) $\frac{1}{1 - sin\,x} + \frac{1}{1 + sin\,x} = \boxed{2sec^2 x}$

$\frac{(1+\sin x)}{(1+\sin x)}\left(\frac{1}{1-\sin x}\right) + \frac{1}{1+\sin x}\left(\frac{1-\sin x}{1-\sin x}\right) = \frac{2}{\cos^2 x}$

$\frac{1+\sin x}{1-\cancel{\sin x}+\cancel{\sin x}-\sin^2 x} \qquad \frac{1-\sin x}{1-\sin^2 x}$

$\frac{1+\sin x}{1-\sin^2 x}$

$\frac{1+\cancel{\sin x}+1-\cancel{\sin x}}{1-\sin^2 x} = \frac{2}{\underbrace{1-\sin^2 x}} = \frac{2}{\cos^2 x} = \boxed{2\sec^2 x}$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x = 1 - \sin^2 x$

f) $\frac{sin^2 x}{1 - cos\,x} = 1 + cos\,x$

g) $\sec x + \tan x = \dfrac{1}{\sec x - \tan x}$

$\sec x + \tan x$

$\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}$

$\dfrac{1+\sin x}{\cos x} = \dfrac{1}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}$

$\dfrac{1}{\frac{1-\sin x}{\cos x}}$ $\quad 1 \div \dfrac{1-\sin x}{\cos x}$

$1 * \dfrac{\cos x}{1-\sin x}$

$\dfrac{1+\sin x}{\cos x} \times \dfrac{\cos x}{1-\sin x}$ ✓

$1-\sin^2 x = \cos^2 x$

true

h) $\dfrac{\cos^2 x + 1 + \sin x}{\cos^2 x + 3} = \dfrac{1+\sin x}{2+\sin x}$

3.) (10 pts total, 5 pts each) Write each expression as a single trigonometric function.

$A = 4x \quad B = 3x$

a) $\sin(4x)\sin(3x) + \cos(4x)\cos(3x)$

$\sin A \sin B + \cos A \cos B$

$\pm \quad \mp$

$\cos(A - B)$

$\cos(4x - 3x) = \boxed{\cos x}$

b) $\sin x \cos(8x) - \cos x \sin(8x)$

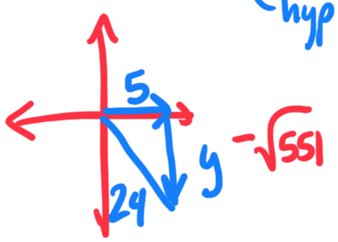
4.) (10 pts total, 5 pts each) Use double angle identities to solve each of the following.

adj ±

a) $If\ \cos x = \frac{5}{24}\ and\ \sin x < 0,\ find\ \tan(2x)$

hyp

$\tan(2x) = \dfrac{2\tan x}{1-\tan^2 x}$



$5^2 + y^2 = 24^2$

$25 + y^2 = 576$

$-25 \qquad -25$

$\sqrt{y^2} = \sqrt{551}$

$y = \sqrt{551}$

$\cos x = \frac{5}{24}$

$\sin x = \frac{-\sqrt{551}}{24}$

I hate you, Nate!!

$\tan x = \dfrac{\frac{-\sqrt{551}}{\cancel{24}}}{\frac{5}{\cancel{24}}}$

$\tan x = \frac{-\sqrt{551}}{5}$

$= \boxed{\dfrac{2\left(\frac{-\sqrt{551}}{5}\right)}{1-\left(\frac{-\sqrt{551}}{5}\right)^2}}$

b) $\sin 15° \cos 15°$