MTH-PT Trigonometry Session 23 4/20

$\sin 15^\circ \cos 15^\circ$

$45 - 30 = 15$

$\theta = 45 \quad \phi = 30$

$\sin 15 = \sin 45 - 30$

$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$

$(\sin 45)(\cos 30) - (\cos 45)(\sin 30)$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos(15)$

$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

$= (\cos 45)(\cos 30) + (\sin 45)(\sin 30)$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15 \cos 15$

$\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) = \frac{6-2}{16} = \frac{4}{16} = \boxed{\frac{1}{4}}$

b) $\sin x \cos(8x) - \cos x \sin(8x)$

$\sin A \cos B - \cos A \sin B$
$\theta \quad \phi \quad \theta \quad \phi$

$A = x \quad B = 8x$

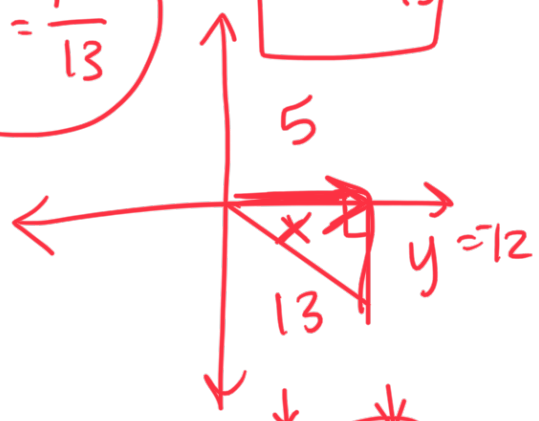
$\sin \theta - \phi$

$\sin x - 8x = \boxed{\sin -7x}$

4.) (10 pts total, 5 pts each) Use double angle identities to solve each of the following.

a) $If \cos x = \frac{5}{13}$ and $\sin x < 0$ find ~~$\tan(2x)$~~ $\cos(2x)$

$\sin x = \frac{-12}{13}$



Pythagorean

$a^2 + b^2 = c^2$

$5^2 + b^2 = 13^2$

$25 + b^2 = 169$

$-25 \quad -25$

$\sqrt{b^2} = \sqrt{144}$

$b = 12$

Double angle

$2\cos^2\theta - 1$

$2\left(\frac{5}{13}\right)^2 - 1$

$2\left(\frac{25}{169}\right) - 1$

$\frac{50}{169} - 1$

$\frac{50}{169} - \frac{169}{169}$

$\boxed{\frac{-119}{169}}$

b) $\sin 15° \cos 15°$

$30 \to 15 \quad \sin 15 = \sin \frac{30}{2}$

$\frac{30}{2}$

$\sin 15 = \sqrt{\frac{1 - \cos 2(15)}{2}}$

$\sqrt{\frac{1 - \cos 30}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$

$\sqrt{\frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{2}} =$

$\sqrt{\frac{\frac{2 - \sqrt{3}}{2}}{2}}$

$\sqrt{\frac{2 - \sqrt{3}}{4}} = \boxed{\frac{\sqrt{2 - \sqrt{3}}}{2}}$

$\cos 15 = \sqrt{\frac{1+\cos 30}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}} \quad \sqrt{\frac{2+\sqrt{3}}{4}} \quad \frac{\sqrt{2+\sqrt{3}}}{2}$

5.) (10 pts total, 2 pts each) Find the exact values of each problem

a) $cos\ 15°$

$\frac{\sqrt{2+\sqrt{3}}}{2}$

$\sin 15 \quad \cos 15$

$\left(\frac{\sqrt{2-\sqrt{3}}}{2}\right)\left(\frac{\sqrt{2+\sqrt{3}}}{2}\right)$

$\frac{\left(\sqrt{2-\sqrt{3}}\right)\left(\sqrt{2+\sqrt{3}}\right)}{4}$

b) $tan\ 202.5°$

6.) (5 pts total) Write the product as a sum or difference.

$4 \leftarrow \boxed{\quad}$ $4sin\ (3x)\ sin\ (4x)$

$\sin\theta \sin\phi = \frac{\cos(\theta-\phi) - \cos(\theta+\phi)}{2}$

$\theta = 3x \qquad \phi = 4x$

$\frac{2}{4}\left(\cos(3x-4x) - \cos(3x+4x)\right)$

$\frac{1}{2}\left(\cos(-x) - \cos(7x)\right)$

Trigonometry Function Identities

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Indentities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\csc^2\theta - \cot^2\theta = 1$$

Even/Odd Indentities

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\frac{\pi}{2} \text{ radians} = 90°$$

Sum/Difference Indentities

$\sin\theta - \phi$

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$

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$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$$

$\sin = \frac{-12}{13}$

Double Angle Identities

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$\cos = \frac{5}{13}$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half Angle Indentities

$$\sqrt{\sin^2\theta} = \sqrt{\frac{1 - \cos(2\theta)}{2}} \qquad \sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$$

$$\sqrt{\cos^2\theta} = \sqrt{\frac{1 + \cos(2\theta)}{2}} \qquad \cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}}$$

$$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

$4\sin(3x)\sin(4x)$

Sum to Product of Two Angles

$$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta + \phi}{2}\right)\cos\left(\frac{\theta - \phi}{2}\right)$$

$$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta + \phi}{2}\right)\sin\left(\frac{\theta - \phi}{2}\right)$$

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta + \phi}{2}\right)\cos\left(\frac{\theta - \phi}{2}\right)$$

$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta + \phi}{2}\right)\sin\left(\frac{\theta - \phi}{2}\right)$$

Product to Sum of Two Angles

$$\sin\theta\sin\phi = \frac{[\cos(\theta - \phi) - \cos(\theta + \phi)]}{2}$$

$$\cos\theta\cos\phi = \frac{[\cos(\theta - \phi) + \cos(\theta + \phi)]}{2}$$

$$\sin\theta\cos\phi = \frac{[\sin(\theta + \phi) + \sin(\theta - \phi)]}{2}$$

$$\cos\theta\sin\phi = \frac{[\sin(\theta + \phi) - \sin(\theta - \phi)]}{2}$$