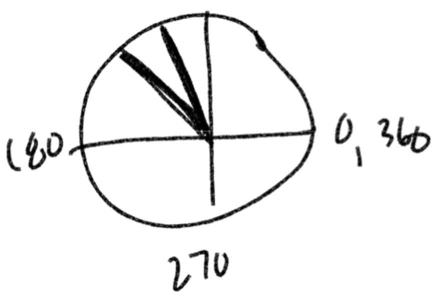
MTH-PT Trigonometry Session 22 4/18

Sum & difference

$\sin 255^\circ$

$\sin(135+120)$



$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$(\sin 135)(\cos 120) + (\cos 135)(\sin 120)$

$\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) \quad \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$-\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

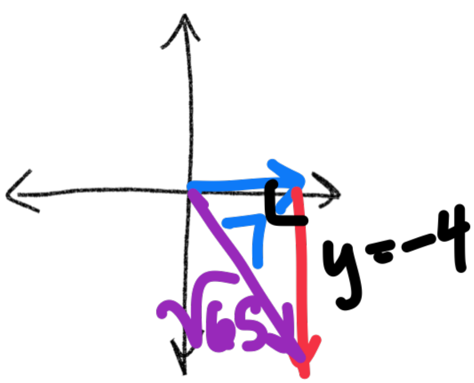
If [A $\cos x = \frac{7}{\sqrt{65}}$] B $\sin x < 0$ C find $\sin 2x$

Make a triangle

sin is $\boxed{\ominus}$

use double angle

$\sin 2x = 2\sin x\cos x$



$\cos x = \frac{7}{\sqrt{65}}$

⊕ right

$\cos = \frac{adj}{\boxed{hyp}} = \frac{7}{\boxed{\sqrt{65}}}$

pythagorean

$7^2 + y^2 = (\sqrt{65})^2 \quad 49 + y^2 = 65 \quad \sqrt{y^2} = \sqrt{16}$

$-49 \qquad -49$

$y = -4$

Trigonometry Function Identities

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Indentities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

ID PROOFS

Quiz 19

Pythagorean Identities

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\csc^2\theta - \cot^2\theta = 1$$

Even/Odd Indentities

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta$$

periodicity

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\frac{\pi}{2} \text{ radians} = 90°$$

Sum/Difference Indentities

1

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$

$+ \rightarrow +$

$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$+ \rightarrow -$

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$$

Quiz 20

Double Angle Identities

2

Q21

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half Angle Indentities

3

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Quiz 21

Sum to Product of Two Angles

4

Q22

$$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta + \phi}{2}\right)\cos\left(\frac{\theta - \phi}{2}\right)$$

$$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta + \phi}{2}\right)\sin\left(\frac{\theta - \phi}{2}\right)$$

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta + \phi}{2}\right)\cos\left(\frac{\theta - \phi}{2}\right)$$

$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta + \phi}{2}\right)\sin\left(\frac{\theta - \phi}{2}\right)$$

Product to Sum of Two Angles

5

$$\sin\theta\sin\phi = \frac{[\cos(\theta - \phi) - \cos(\theta + \phi)]}{2}$$

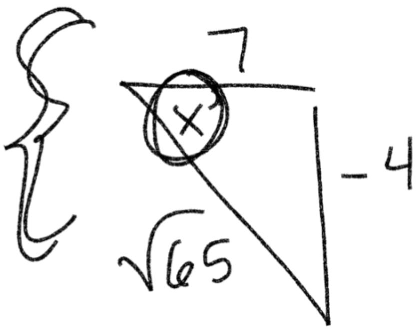
$$\cos\theta\cos\phi = \frac{[\cos(\theta - \phi) + \cos(\theta + \phi)]}{2}$$

$$\sin\theta\cos\phi = \frac{[\sin(\theta + \phi) + \sin(\theta - \phi)]}{2}$$

Q22

$$\cos\theta\sin\phi = \frac{[\sin(\theta + \phi) - \sin(\theta - \phi)]}{2}$$

Find $\sin 2x$ $\quad 2\sin x \cos x$



$\cos x = \frac{7}{\sqrt{65}}$ $\quad \sin x = \frac{opp}{hyp} = \frac{-4}{\sqrt{65}}$

$$2\left(\frac{-4}{\sqrt{65}}\right)\left(\frac{7}{\sqrt{65}}\right)$$

$$\boxed{\frac{-56}{65}}$$

$$\underbrace{\text{If } \sin x = \frac{2}{\sqrt{29}}}_{\text{Draw triangle}} \quad \underbrace{\cos x > 0}_{\cos x \oplus} \quad \underbrace{\text{find } \sin \frac{x}{2}}_{\text{Look up Half angle}}$$

$$\sin\left(\frac{A}{2}\right) = \boxed{\pm\sqrt{\frac{1-\cos A}{2}}}$$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$\sin^2\theta = \frac{1-\cos 2(\theta)}{2}$$

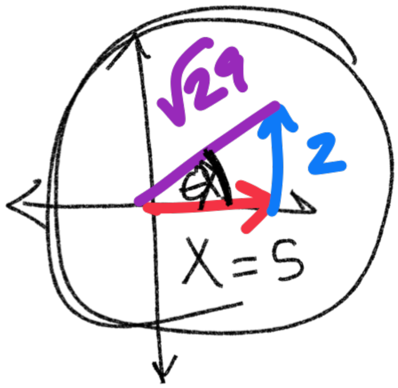
$$\sin\theta = \pm\sqrt{\frac{1-\cos 2\theta}{2}}$$

or

If $\sin x = \dfrac{2}{\sqrt{29}}$ ← up

$\cos x > 0$ find $\sin \dfrac{x}{2}$

+ right



$A^2 + B^2 = c^2$

$2^2 + x^2 = (\sqrt{29})^2$

$4 + x^2 = 29$

$-4 \qquad -4$

$\sqrt{x^2} = \sqrt{25}$

$x = 5$

$\cos x = \dfrac{5}{\sqrt{29}}$

$\cos = \dfrac{adj}{hyp}$

$$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}} = \boxed{\pm\sqrt{\frac{1 - \frac{5}{\sqrt{29}}}{2}}}$$

$$\sqrt{\frac{\frac{\sqrt{29}}{\sqrt{29}} - \frac{5}{\sqrt{29}}}{2}}$$

$$\boxed{\sqrt{\frac{\sqrt{29} - 5}{2\sqrt{29}}}}$$

product $\longrightarrow$ sum

$$\cos 3x \cos 5x$$

θ $\qquad$ ϕ

$$\cos\theta\cos\phi = \frac{\cos(\theta-\phi)+\cos(\theta+\phi)}{2}$$

even / odd

$\cos 3x-5x \qquad \cos 3x+5x$

$$\frac{\cos(-2x)+\cos(8x)}{2}$$

$\cos(-\theta)=\cos\theta$

$$\boxed{\frac{\cos(2x)+\cos(8x)}{2}}$$

sum $\longrightarrow$ product

$$\sin \boxed{3x} + \sin \boxed{5x}$$

$$\boxed{\theta} \qquad \boxed{\phi}$$

$$\sin \boxed{\theta} + \sin \boxed{\phi} = 2 \sin\left(\frac{\boxed{\theta}+\boxed{\phi}}{2}\right) \cos\left(\frac{\boxed{\theta}-\boxed{\phi}}{2}\right)$$

$$\sin 3x + \sin 5x = 2 \sin\left(\frac{3x+5x}{2}\right) \cos\left(\frac{3x-5x}{2}\right)$$

$$2 \sin\left(\frac{8x}{2}\right) \cos\left(\frac{-2x}{2}\right)$$

even/odd

At the end, look out for even/odd

$$2 \sin(4x) \cos(-x)$$

$$\boxed{2 \sin(4x) \cos(x)}$$