Calculus & Trigonometry

Chapter 6 Pre-Test

1.) (20 pts total, 5 pts each) Simplify each of the following trigonometric expressions.

a) $(\sin^2 x)(\cot^2 x + 1)$

Convert to sin/cos

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x \left(\frac{\cos^2 x}{\sin^2 x} + 1\right)$

$\cancel{\frac{\sin^2 x \cos^2 x}{\sin^2 x}} + \sin^2 x$

$\cos^2 x + \sin^2 x = \boxed{1}$

b) $(\sin x - \cos x)(\sin x + \cos x)$

c) $\frac{1 - \cos^4 x}{1 + \cos^2 x}$

"Factor"

Difference of squares

$1 - \cos^4 x = (1 + \cos^2 x)(1 - \cos^2 x)$

$\sqrt{1} \to 1 \qquad \sqrt{\cos^4 x} \to \cos^2 x$

$\frac{1 - \cos^4 x}{1 + \cos^2 x} = \frac{\cancel{(1 + \cos^2 x)}(1 - \cos^2 x)}{\cancel{1 + \cos^2 x}}$

$1 - \cos^2 x = \boxed{\sin^2 x}$

$\sin^2 x + \cos^2 x = 1$

$\quad - \cos^2 x \quad - \cos^2 x$

$\sin^2 x = 1 - \cos^2 x$

d) $1 - \frac{\sin^2 x}{1 - \cos x}$

2.) (40 pts total, 5 pts each) Verify each of the following trigonometric identities.

a) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = \boxed{2}$

$(\sin x + \cos x)(\sin x + \cos x)$

$\sin^2 x + \cancel{\sin x \cos x} + \cancel{\sin x \cos x} + \cos^2 x$

$\sin^2 x - \cancel{\sin x \cos x} - \cancel{\sin x \cos x} + \cos^2 x$

$(\sin x - \cos x)(\sin x - \cos x)$

$\underbrace{\sin^2 x + \cos^2 x}_{1} + \underbrace{\sin^2 x + \cos^2 x}_{1} = 2$

b) $\tan x + \cot x = \csc x \sec x$

c) $\dfrac{2 - \sin^2 x}{\cos x} = \sec x + \cos x$

$\dfrac{2 - \sin^2 x}{\cos x} = \dfrac{1}{\cos x} + \dfrac{\cos x}{1} \cdot \dfrac{\cos x}{\cos x}$

$\dfrac{2 - \sin^2 x}{\cos x} = \dfrac{1}{\cos x} + \dfrac{\cos^2 x}{\cos x}$

$\dfrac{2 - \sin^2 x}{\cancel{\cos x}} = \dfrac{1 + \cos^2 x}{\cancel{\cos x}}$

$2 - \sin^2 x = \boxed{1 + \cos^2 x}$

$2 - \sin^2 x = 1 + \cos^2 x$
$+\sin^2 x \qquad +\sin^2 x$

$2 = 1 + \sin^2 x + \cos^2 x$
$-1 \qquad -1$

$1 = \sin^2 x + \cos^2 x$

$\sin^2 x + \cos^2 x = 1$
$-\cos^2 x \qquad -\cos^2 x$
$\sin^2 x = \boxed{1 - \cos^2 x}$

$2 - (1 - \cos^2 x)$

$2 - 1 + \cos^2 x$

$\boxed{1 + \cos^2 x}$

d) $\frac{1}{csc^2 x} + \frac{1}{sec^2 x} = 1$

e) $\frac{1}{1 - sin\,x} + \frac{1}{1 + sin\,x} = 2sec^2 x$

$$\frac{(1+\sin x)}{(1+\sin x)}\frac{1}{(1-\sin x)} + \frac{1}{(1+\sin x)}\frac{(1-\sin x)}{(1-\sin x)} = \frac{2}{\cos^2 x}$$

$$\frac{1+\sin x}{1-\sin^2 x} + \frac{1-\sin x}{1-\sin^2 x} = \boxed{\frac{2}{\cos^2 x}}$$

$$\frac{1+\cancel{\sin x}+1-\cancel{\sin x}}{1-\sin^2 x} = \boxed{\frac{2}{\cos^2 x}}$$

f) $\frac{sin^2 x}{1 - cos\,x} = 1 + cos\,x$

g) $\sec x + \tan x = \dfrac{1}{\sec x - \tan x}$

$1 \div \dfrac{1-\sin x}{\cos x}$

$1 * \dfrac{\cos x}{1-\sin x}$

$\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x} = \dfrac{1}{\dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}}$

$\dfrac{1+\sin x}{\cos x} = \dfrac{1}{\dfrac{1-\sin x}{\cos x}}$

$\dfrac{1+\sin x}{\cos x} \times \boxed{\dfrac{\cos x}{1-\sin x}}$

$\cos^2 x = 1 - \sin^2 x$

$\sin^2 x + \cos^2 x = 1$

$\dfrac{1+\sin x \cdot \cos x}{\cos x \cdot \cos x}$

$\dfrac{\cos x(1+\sin x)}{\cos^2 x}$

h) $\dfrac{\cos^2 x + 1 + \sin x}{\cos^2 x + 3} = \dfrac{1+\sin x}{2+\sin x}$

Difference of squares

$\dfrac{\cos x(1+\sin x)}{1-\sin^2 x}$

$\dfrac{\cos x\,\cancel{(1+\sin x)}}{(1-\sin x)\cancel{(1+\sin x)}}$

$\boxed{\dfrac{\cos x}{1-\sin x}}$

3.) (10 pts total, 5 pts each) Write each expression as a single trigonometric function.

$A = 4x \quad B = 3x$

a) $\sin(4x)\sin(3x) + \cos(4x)\cos(3x)$

$\sin A \sin B + \cos A \cos B$

$\cos A - B$

$\cos(4x - 3x)$

$\boxed{\cos x}$

b) $\sin x \cos(8x) - \cos x \sin(8x)$

$\sin\theta\cos\phi - \cos\theta\sin\phi$

$\sin(x-8x) = \boxed{\sin(-7x)}$

4.) (10 pts total, 5 pts each) Use double angle identities to solve each of the following.

a) $If \cos x = \frac{5}{24}\ and\ \sin x < 0,\ find\ \boxed{\tan(2x)}$

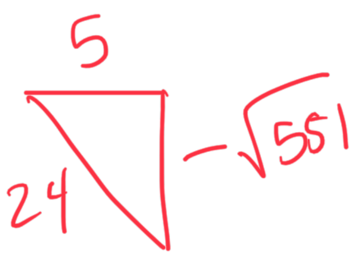
$5^2 + b^2 = 24^2$

$25 + b^2 = 576$
$-25 \qquad -25$

$b^2 = 551$

$b = \sqrt{551}$

$\sin x = \frac{-\sqrt{551}}{24}$



$\frac{2\tan x}{1-\tan^2 x}$

$\frac{2\left(\frac{-\sqrt{551}}{5}\right)}{1-\left(-\frac{\sqrt{551}}{5}\right)^2}$

b) $\sin 15° \cos 15°$

Trigonometry Function Identities

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Indentities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\csc^2\theta - \cot^2\theta = 1$$

Even/Odd Indentities

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta$$

$\sin\theta - \phi$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\frac{\pi}{2} \text{ radians} = 90^\circ$$

Sum/Difference Indentities

$$\boxed{\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi}$$

$+ \sin A \sin B \quad \cos A \cos B$

$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$$

$\theta = A$

$\phi = B$

$\cos(A-B)$

Double Angle Identities

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\frac{2\tan x}{1 - \tan^2 x}$$

Half Angle Indentities

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Sum to Product of Two Angles

$$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta + \phi}{2}\right)\cos\left(\frac{\theta - \phi}{2}\right)$$

$$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta + \phi}{2}\right)\sin\left(\frac{\theta - \phi}{2}\right)$$

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta + \phi}{2}\right)\cos\left(\frac{\theta - \phi}{2}\right)$$

$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta + \phi}{2}\right)\sin\left(\frac{\theta - \phi}{2}\right)$$

Product to Sum of Two Angles

$$\sin\theta\sin\phi = \frac{[\cos(\theta - \phi) - \cos(\theta + \phi)]}{2}$$

$$\cos\theta\cos\phi = \frac{[\cos(\theta - \phi) + \cos(\theta + \phi)]}{2}$$

$$\sin\theta\cos\phi = \frac{[\sin(\theta + \phi) + \sin(\theta - \phi)]}{2}$$

$$\cos\theta\sin\phi = \frac{[\sin(\theta + \phi) - \sin(\theta - \phi)]}{2}$$

5.) (10 pts total, 2 pts each) Find the exact values of each problem

a) $cos\ 15°$

b) $tan\ 202.5°$

6.) (5 pts total) Write the product as a sum or difference.

$$4sin\ (3x)\ sin\ (4x)$$

7.) (5 pts total) Write the expression as a product.

$$\cos(8x) + \cos(3x)$$