MTH-PT Trigonometry Session 21 4/15

$\sin \frac{-\pi}{12}$ $\quad \frac{2\pi}{12} - \frac{3\pi}{12} = \frac{-\pi}{12}$

$\sin\left(\left(\frac{\pi}{6}\right) - \left(\frac{\pi}{4}\right)\right)$

Difference

$\frac{\pi}{6} + \left(\frac{-\pi}{4}\right)$

sum

$\sin(A-B) = \sin A\cos B - \cos A \sin B$

$\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

$\cos(-15^\circ)$

$\cos(30-45)$

$\cos(A+B) = \cos A\cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$(\cos 30)(\cos 45) + (\sin 30)(\sin 45)$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

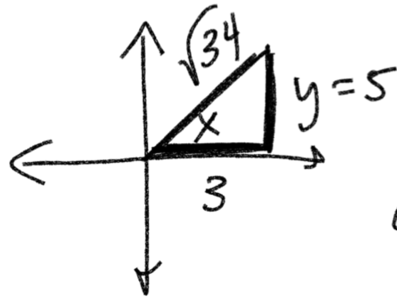
$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

Double Angle

If $\cos x = \dfrac{+3}{\sqrt{34}}$ ← adj, ← hyp, and $\sin x > 0$

find $\cos 2x$

1.) Draw triangle



from sheet

$\cos 2x = \cos^2 x - \sin^2 x$

or

$1 - 2\sin^2 x$

or

$2\cos^2 x - 1$

$\cos x = \dfrac{adj}{hyp}$

$\cos x = \boxed{\dfrac{3}{\sqrt{34}}}$

$\sin x = \dfrac{5}{\sqrt{34}}$

Pythagorean Theorem

$3^2 + y^2 = (\sqrt{34})^2$

$9 + y^2 = 34$

$-9 \qquad -9$

$\sqrt{y^2} = \sqrt{25}$

$y = 5$

$\cos 2x = (\cos x)^2 - (\sin x)^2$

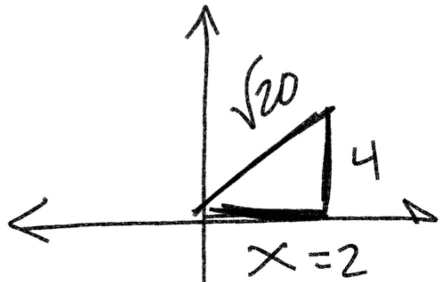
$\downarrow \qquad \downarrow$

$\left(\dfrac{3}{\sqrt{34}}\right)^2 - \left(\dfrac{5}{\sqrt{34}}\right)^2$

$\dfrac{9}{34} - \dfrac{25}{34} = \dfrac{-16}{34} = \boxed{\dfrac{-8}{17}}$

If $\sin x = \frac{4}{\sqrt{20}}$ and $\underbrace{\cos x}_{+} > 0$ Find $\sin 2x$

$$\underbrace{\sin 2x = 2\sin x \cos x}$$



Pythagorean Theorem

$$x^2 + 4^2 = (\sqrt{20})^2$$

$$x^2 + 16 = 20$$
$$-16 \quad -16$$
$$\sqrt{x^2} = \sqrt{4}$$

$x = 2$

$\cos x = \frac{2}{\sqrt{20}}$

$$\sin 2x = 2\left(\frac{4}{\sqrt{20}}\right)\left(\frac{2}{\sqrt{20}}\right) = \frac{16 \div 4}{20 \div 4}$$

$$\boxed{\frac{4}{5}}$$

Half Angle

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

Half Angle

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$\sin 30^\circ = \frac{1}{2}$

$\downarrow$

$\sin\left(\frac{60}{2}\right)$

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$A=60$

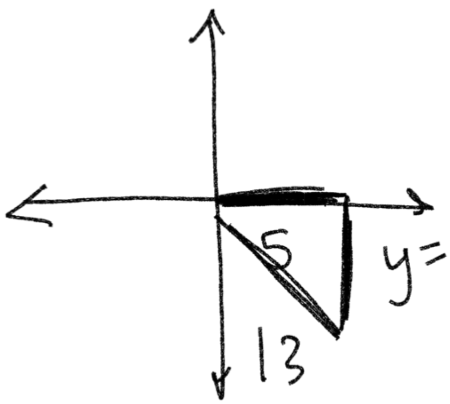
$$\pm\sqrt{\frac{1-\cos 60}{2}}$$

$$\pm\sqrt{\frac{1-\frac{1}{2}}{2}} = \pm\sqrt{\frac{\frac{1}{2}}{2}}$$

$$\pm\sqrt{\frac{1}{4}} = \boxed{\pm\frac{1}{2}}$$

If $\cos x = \frac{5}{13}$ $\quad \sin x < 0 \quad$ find $\left[\sin \frac{x}{2}\right]$

θ



$y = -12$

$5^2 + y^2 = 13^2$

$25 + y^2 = 169$

$-25 \qquad -25$

$\sqrt{y^2} = \sqrt{144}$

$y = 12$

$\sin \frac{A}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$

$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \frac{5}{13}}{2}}$

$\pm\sqrt{\frac{\frac{13}{13} - \frac{5}{13}}{2}}$

$\pm\sqrt{\frac{\frac{8}{13}}{2}} = \pm\sqrt{\frac{8}{26}}$

$\boxed{\pm\sqrt{\frac{4}{13}}} = \boxed{\pm\frac{2}{\sqrt{13}}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$

$\boxed{\pm\frac{2\sqrt{13}}{13}}$

Product-to-Sum Identities

"Sheet"

$$\left[\cos A \cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)\right]$$

$$4\left[\cos(3x)\cos(2x)\right]$$

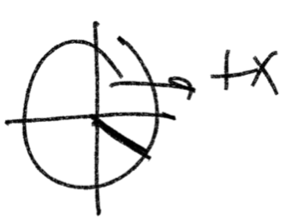
$$4\left(\frac{1}{2}\left(\cos(3x+2x) + \cos(3x-2x)\right)\right)$$

$$\boxed{2\left(\cos(5x) + \cos x\right)}$$

$$(\cos 30^\circ)(\cos 60^\circ)$$

$$\frac{\sqrt{3}}{2} * \frac{1}{2} = \frac{\sqrt{3}}{4}$$

$$(\cos 30^\circ)(\cos 60^\circ) = \frac{1}{2}\left(\cos(30+60) + \cos(30-60)\right)$$



$$\frac{1}{2}\left(\cos 90 + \cos(-30)\right)$$

$$\frac{1}{2}\left(0 + \frac{\sqrt{3}}{2}\right)$$

$$\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4}$$