MTH-PT Trigonometry Session 21 4/13

If $\underbrace{\cos x = \frac{3}{\sqrt{34}}}$ and $\sin x > 0$ find $\cos 2x$

$\cos 2x = \cos^2 x - \sin^2 x$

$1 - 2\sin^2 x$

$\underbrace{2\cos^2 x - 1}$

$2(\cos x)^2 - 1$

$\downarrow$

$2\left(\frac{3}{\sqrt{34}}\right)^2 - 1$

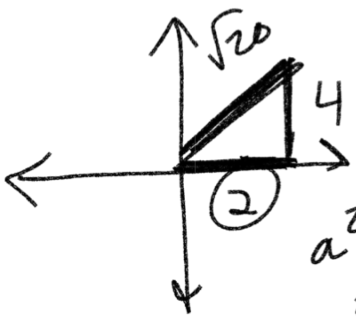
$2\left(\frac{3}{\sqrt{34}}\right)\left(\frac{3}{\sqrt{34}}\right) - 1$

$2\left(\frac{9}{34}\right) - 1 = \frac{18}{34} - 1 = \frac{18}{34} - \frac{34}{34} = \frac{-16}{34} \; -\frac{8}{17}$

$+$

If $\sin x = \frac{4}{\sqrt{20}}$ and $\cos x > 0$ find $\sin 2x$

$\sin 2x = 2\sin x \cos x$



1.) Make triangle

$a^2 + b^2 = c^2 \rightarrow$ 1.b)

2.) find $\sin x$ $\boxed{\cos x}$

3.) plug in

$4^2 + b^2 = (\sqrt{20})^2$

$16 + b^2 = 20$

$-16 \qquad -16$

$\sqrt{b^2} = \sqrt{4} \quad b = 2$

$\cos x = \frac{2}{\sqrt{20}}$

$\sin 2x = 2\left(\frac{4}{\sqrt{20}}\right)\left(\frac{2}{\sqrt{20}}\right)$

$\frac{16}{20} = \boxed{\frac{4}{5}}$

Evaluate $\boxed{\dfrac{2\tan 15^\circ}{1-\tan^2 15^\circ}} = \tan 2(15^\circ)$

$\tan 30^\circ$

$\tan 2A = \boxed{\dfrac{2\tan A}{1-\tan^2 A}}$

$A = 15^\circ$

$$\tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{\frac{1}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}}$$

$$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

Half Angle

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$\sin 30^\circ = \frac{1}{2}$

$\sin \frac{60}{2} = \frac{1}{2}$

$A = 60$

$$\sin \frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\sin \frac{60}{2} = \pm\sqrt{\frac{1-\cos 60}{2}}$$

$$\sin \frac{60}{2} = \pm\sqrt{\frac{1-\frac{1}{2}}{2}} = \pm\sqrt{\frac{\frac{1}{2}}{2}} = \pm\sqrt{\frac{1}{4}}$$

$$\pm\frac{1}{2}$$

If $\cos x = \frac{5}{13}$ $\sin x < 0$ find $\sin\frac{x}{2}$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\pm\sqrt{\frac{1-\frac{5}{13}}{2}}$$

$$\pm\sqrt{\frac{\frac{13}{13}-\frac{5}{13}}{2}} = \pm\sqrt{\frac{\frac{8}{13}}{2}} = \pm\sqrt{\frac{8}{26}} = \pm\sqrt{\frac{4}{13}}$$

$$\boxed{-\sqrt{\frac{4}{13}}}$$

$$-\frac{\sqrt{4}}{\sqrt{13}} = -\frac{2}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = \boxed{\frac{-2\sqrt{13}}{13}}$$

Product-to-sum Identities

$$\boxed{\cos A\cos B} = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$4\left[\cos(3x)\cos(2x)\right]$$

$A = 3x$ $B = 2x$ $4\left(\frac{1}{2}\left[\cos(3x+2x) + \cos(3x-2x)\right]\right)$

$$\boxed{2\left[\cos 5x + \cos x\right]}$$

$(\cos 30°)(\cos 60°)$ $\quad A = 30 \quad B = 60$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4}$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\cos 30 \cos 60 = \frac{1}{2}\left[\cos(30+60) + \cos(30-60)\right]$$

$$\frac{1}{2}\left[\cos 90 + \cos -30\right]$$

$$\frac{1}{2}\left[0 + \frac{\sqrt{3}}{2}\right] : \frac{1}{2}\,\frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{3}}{4}}$$

$$\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

$$\sin A \cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$

$-3 \sin 2x \sin 4x$

$A = 2x \quad B = 4x \quad -3\left[\frac{1}{2}\left(\cos(2x-4x) - \cos(2x+4x)\right)\right]$

$$\boxed{-\frac{3}{2}\left(\cos(-2x) - \cos(6x)\right)}$$

sum-to-product

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\sin 30 + \sin 60$ $\quad A = 30 \quad B = 60$

$\frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

$2\sin\left(\frac{30+60}{2}\right)\cos\left(\frac{30-60}{2}\right)$

$2(\sin 45)(\cos -15)$

$\cos(-15) = \cos(45-60)$

$\cos A-B = \cos A \cos B + \sin A \sin B$

$\cos 45 \cos 60 + \sin 45 \sin 60$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{\frac{1+\sqrt{3}}{2}}$

$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

$2(\sin 45)(\cos -15)$

$2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)$ $\quad 2\left(\frac{2+\sqrt{12}}{8}\right)$ $\quad \frac{2+\sqrt{12}}{4} = \frac{2+2\sqrt{3}}{4}$