MTH-PT Trigonometry Session 20 4/11

Sum and Difference Identities

<u>NO MEMORIZING!!</u> of Identities

$\cos 105°$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$60° + 45° = 105°$

$150° - 45° = 105°$

$\cos 105°$

$$\cos(\overset{A}{60} + \overset{B}{45}) = (\cos 60)(\cos 45) - (\sin 60)(\sin 45)$$

$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

$\cos 105$

$$\cos(\overset{A}{150} - \overset{B}{45}) =$$

$$(\cos 150)(\cos 45) + (\sin 150)(\sin 45)$$

$$\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

$\cos\left(\frac{7\pi}{12}\right)$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\frac{4\pi}{12} + \frac{3\pi}{12} = \frac{7\pi}{12}$ ✓

$\downarrow \quad \downarrow$

$\frac{\pi}{3} + \frac{\pi}{4}$

$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$\cos A \cos B - \sin A \sin B$

$\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

$\frac{7\cancel{\pi}}{12_{\div 12}} * \frac{180^{\div 12}}{\cancel{\pi}}$

$\frac{7}{1} * \frac{15}{1} = 105^\circ$

Sum & difference of sine

Sum: $\sin(A+B) = \sin A \cos B + \cos A \sin B$

Difference: $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin 75^\circ = \sin(\overset{A}{45} + \overset{B}{30})$

$\sin A \cos B + \cos A \sin B$

$(\sin 45)(\cos 30) + (\cos 45)(\sin 30)$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

Sum and difference of tangent $\qquad \tan = \frac{\sin}{\cos}$

Sum: $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

Difference $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\boxed{\tan 45^\circ = 1}$

$\tan 165^\circ$

$$\tan(120+45) = \frac{\tan 120 + \tan 45}{1 - (\tan 120)(\tan 45)}$$

$\tan = \frac{\sin}{\cos}$

$$\frac{\dfrac{\sin 120}{\cos 120} + \dfrac{\sin 45}{\cos 45}}{1 - \left(\dfrac{\sin 120}{\cos 120}\right)\left(\dfrac{\sin 45}{\cos 45}\right)}$$

$$\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$$

$$\frac{\sqrt{3}}{2} \div -\frac{1}{2}$$

$$\frac{\sqrt{3}}{\cancel{2}} * \frac{-\cancel{2}}{1}$$

Keep, change, flip.

$\sqrt{3} * -1 = -\sqrt{3}$

$$\frac{\dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} + \cancel{\dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}}^{\,1}}{1 - \left(\dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)(1)}$$

$$\frac{-\sqrt{3}+1}{1-(-\sqrt{3})} = \boxed{\frac{-\sqrt{3}+1}{1+\sqrt{3}}}$$

Double and Half Angle Identities

$$\sin 60^\circ = \sin 2(30^\circ)$$

Double Angle Identity

on the sheet

$$[\sin 2x = 2\sin x\cos x]$$

$$\sin 60^\circ = \sin 2(30) = 2(\sin 30)(\cos 30)$$

$$\frac{\sqrt{3}}{2}$$

$$\cancel{2}\left(\frac{1}{\cancel{2}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$$

$$[\sin 2x = 2\sin x\cos x]$$

on sheet!

$$\cos 2x = [\boxed{\cos^2 x} - \sin^2 x]$$

or

$$1 - 2\sin^2 x$$

or

$$2\cos^2 x - 1$$

$$[\sin^2 x + \cos^2 x = 1]$$

$$-\sin^2 x \qquad -\sin^2 x$$

$$\cos^2 x = \boxed{1 - \sin^2 x}$$

$$1 - \underbrace{\sin^2 x - \sin^2 x}$$

$$1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

If $\sin x = \frac{1}{\sqrt{5}}$ and $[\cos x < 0]$, Find $\sin 2x$

opp(y)

hyp

$$\sin x = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r}$$

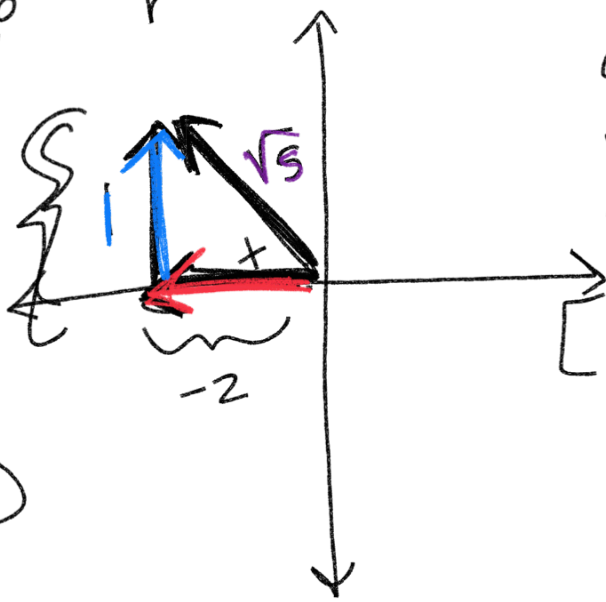
opp → 1

hyp → $\sqrt{5}$

adj → ?

$\cos x = \frac{-2}{\sqrt{5}}$

$\sin x = \frac{1}{\sqrt{5}}$



Pythagorean Theorem

$$a^2 + b^2 = c^2$$

$$a^2 + (1)^2 = (\sqrt{5})^2$$

$$[a^2 + 1 = 5]$$

$$-1 \quad -1$$

$$\sqrt{a^2} = \sqrt{4}$$

$$a = \pm 2$$

$$\sin 2x = 2 \sin x \cos x$$

$$2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{-2}{\sqrt{5}}\right) = \boxed{\frac{-4}{5}}$$