MTH-PT Trigonometry session 20 4/10

sum and difference identities

$\cos 105^\circ$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(45+60)$

$$= \cos 45 \cos 60 - \sin 45 \sin 60$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

$\sin 165^\circ$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin(45+120)$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin 45 \cos 120 + \cos 45 \sin 120$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

$\tan 165^\circ$

$\tan(120+45)$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan 120+45 = \frac{\tan 120 + \tan 45}{1 - \tan 120 \tan 45}$$

$$\tan 120+45 = \frac{\frac{\sin 120}{\cos 120} + \frac{\sin 45}{\cos 45}}{1 - \left(\frac{\sin 120}{\cos 120}\right)\left(\frac{\sin 45}{\cos 45}\right)}$$

$$= \frac{\frac{\frac{\sqrt{3}}{\cancel{2}}}{\frac{1}{\cancel{2}}} + \cancelto{1}{\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}}}{1 - \left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)\cancelto{1}{\left(\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right)}}$$

$$\boxed{\frac{\sqrt{3} + 1}{1 - \sqrt{3}}}$$

Double and Half Angles

$\sin 60° = \sin 2(30°)$

$\sin 2(30°) = 2 \sin 30° \cos 30°$

Double Angle Identity

$\sin 2X = 2 \sin X \cos X$

$\sin 60° = \sin 2(30°) = 2 \sin 30° \cos 30°$

$\cancel{2}\left(\frac{1}{\cancel{2}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$1 - 2\sin^2 A$$

$$2\cos^2 A - 1$$

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

If $\sin x = \frac{1}{\sqrt{5}}$ and $\boxed{\cos x < 0}$ find

-2 $\quad$ $\sin 2x$

$$\sin x = \frac{opp}{hyp} = \frac{y}{r}$$

pythagorean theorem

$$a^2 + b^2 = c^2$$

$$\downarrow$$

$$(1)^2 + b^2 = (\sqrt{5})^2$$

$$1 + b^2 = 5$$

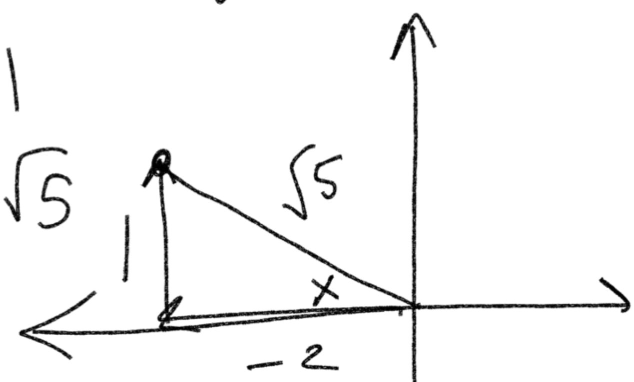
$$-1 \qquad\qquad -1$$

$$\sqrt{b^2} = \sqrt{4}$$

$$b = \pm 2$$

opp = 1

hyp = $\sqrt{5}$



$$\cos x = \frac{adj}{hyp} = \frac{-2}{\sqrt{5}}$$

$$\sin x = \frac{1}{\sqrt{5}}$$

Find $\sin 2X$

$$\sin 2x = 2 \sin x \cos x$$

$$= 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{-2}{\sqrt{5}}\right) = \boxed{\frac{-4}{5}}$$

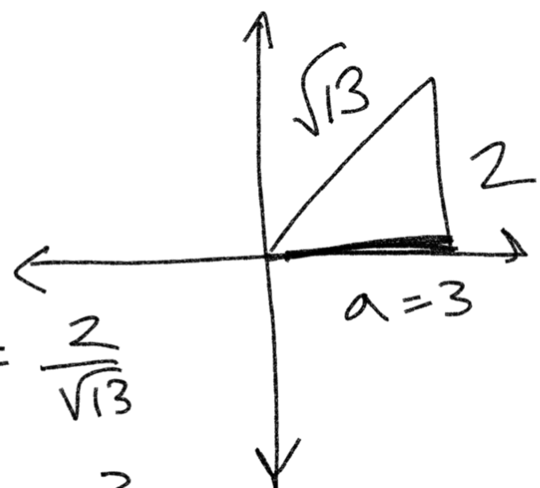
If $\sin x = \frac{2}{\sqrt{13}}$ and $\cos x > 0$ find $\cos(2x)$

$+$ $+3$

1.) Build triangle

2.) Find both x and y

3.) Find sin x and cos x



$$2^2 + a^2 = (\sqrt{13})^2$$

$$4 + a^2 = 13$$

$$-4 \qquad -4$$

$$\sqrt{a^2} = \sqrt{9}$$

$$a = \pm 3$$

$$\sin x = \frac{2}{\sqrt{13}}$$

$$\cos x = \frac{3}{\sqrt{13}}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\left(\frac{3}{\sqrt{13}}\right)^2 - \left(\frac{2}{\sqrt{13}}\right)^2$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\frac{9}{13} - \frac{4}{13} = \boxed{\frac{5}{13}}$$