MTH-PT Trigonometry Session 2 1/30

$6^{\frac{1}{2}} = \pm\sqrt{6}$

$$81^{3/2} = \left(\sqrt{81}\right)^3 = (\pm 9)^3 \quad \boxed{= \pm 729}$$

exponent

root

$(+9)^3$ $(-9)^3$

729 -729

$$36^{k-3} = 216^{3k}$$

$$(6^2)^{k-3} = (6^3)^{3k}$$

$$6^{2k-6} = 6^{9k}$$

$$2k - 6 = 9k$$
$$-2k \qquad -2k$$

$$\frac{-6}{7} = \frac{7k}{7}$$

$$\boxed{k = \frac{-6}{7}}$$

$$64^{3k-2} = 16^{-3k}$$

$$(4^3)^{3k-2} = (4^2)^{-3k}$$

$$4^{\boxed{9k-6}} = 4^{\boxed{-6k}}$$

$$9k - 6 = -6k$$
$$-9k \qquad -9k$$

$$\frac{-6}{-15} = \frac{-15k}{-15}$$

$$k = \frac{6 \div 3}{15 \div 3} = \boxed{\frac{2}{5}}$$

# Graphing Exponents

$y = \frac{1}{3}\left(\frac{1}{2}\right)^x$ ← exponent

$\frac{1}{3}$ ← amplitude

$\frac{1}{2}$ ← base

$y = ab^x$

$a$ ← "slope" amplitude

$y = ab^{(x-h)} + k$

$(h, k) \rightarrow$ denoted translation

$y = \frac{1}{3}\left(\frac{1}{2}\right)^x$

exponent = 0

$x = 0$

$y = \frac{1}{3}\left(\frac{1}{2}\right)^0$

$\frac{1}{3}(1) = \frac{1}{3}$

$(0, \frac{1}{3})$

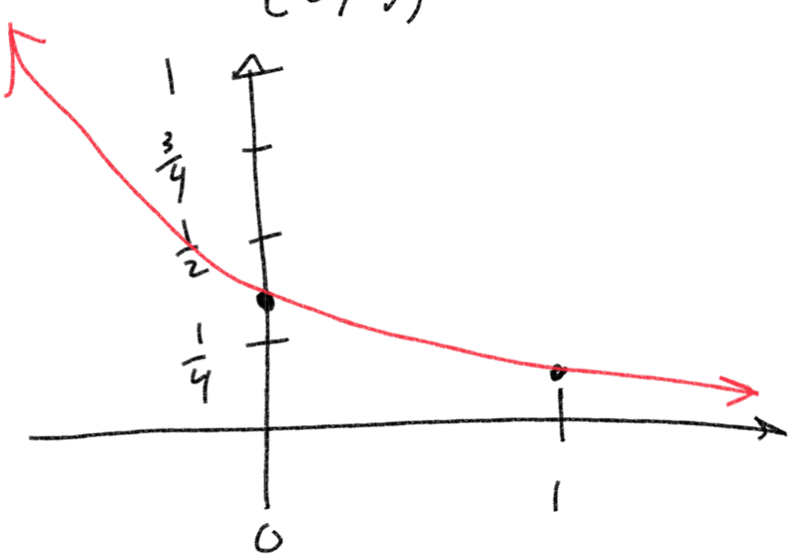
exponent = 1

$x = 1$

$y = \frac{1}{3}\left(\frac{1}{2}\right)^1$

$\frac{1}{3}\left(\frac{1}{2}\right) = \frac{1}{6}$

$(1, \frac{1}{6})$




$y = \frac{1}{3}\left(\frac{1}{2}\right)^x$

$\frac{1}{2}$ ← base

base < 1
decreasing

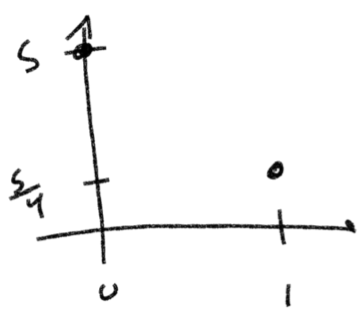
base > 1 increasing

1.) $y = 5\left(\frac{1}{4}\right)^{x}$ $<1$ decreasing

increasing decreasing

$x = 0$ $y = 5$

$x = 1$ $y = \frac{5}{4}$




2.) $y = \frac{1}{3}(2)^{x}$ $>1$ increasing

increasing decreasing

$x = 0$ $y = \frac{1}{3}$

$x = 1$ $y = \frac{2}{3}$

$\frac{1}{3}(2)^{0} = \frac{1}{3}(1) = \frac{1}{3}$

## Compound Interest

$$A = A_0\left(1 + \frac{r}{n}\right)^{nt}$$

$A$ = total amount

$A_0$ = initial amount

$r$ = rate (decimal)

$n$ = compound frequency

$t$ = time (years)

Daily $n = 365$

monthly $n = 12$

quarterly $n = 4$

$A_0 = \$2{,}000$

$r = 8\%$

compound semi-annually $n = 2$

$t = 6$ years

$$A = 2000\left(1 + \frac{0.08}{2}\right)^{2(6)}$$

$$2000(1.04)^{12}$$

$$\boxed{\$3202.06}$$

Compounded Continuously

$n = \infty$

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = 2.7172\ldots\ldots = e$$

"natural number"

Continuous Compounding Interest

$A = Pe^{rt}$

$P$ = principal or initial amount

$e = e$ (natural number)

$r$ = rate (decimal)

$t$ = time (years)

initial amount = \$8,615

$r = 2\%$

compounded continuously $\rightarrow$ $A = Pe^{rt}$

$t = 11$ years

$A = (8{,}615)\,e^{(0.02)(11)}$

$(8{,}615)\,e^{0.22}$

$\boxed{= 10{,}734.95}$

$$A = A_0\left(1+\frac{r}{n}\right)^{nt}$$

$A_0 = 1{,}000$ $t = 1$

$r = 5\%$ $(0.05)$

$n = 1$ yearly ✓ $\longrightarrow$ \$1050

$1000\left(1+\frac{0.05}{1}\right)^{(1)(1)}$

$n = 2$ semi-yearly ✓ $\longrightarrow$ \$1,050.62

$n = 12$ monthly ✓ $\longrightarrow$ \$1,051.16

$1000\left(1+\frac{0.05}{12}\right)^{12(1)}$

$n = 365$ daily ✓ $\longrightarrow$ \$1,051.267

secondly ✓ $(365)(24)(60)(60) = 1{,}051.27109$

$n = 31{,}536{,}000$

$n = \infty$ $A = Pe^{rt}$

$1000\,e^{(0.05)(1)} = 1{,}051.27109\ldots$

$P = \$500{,}000$ Goal $= \$2{,}500{,}000$

$r = 6\%$ $t = ?$

$A = Pe^{rt}$

$\downarrow$

$\$2{,}500{,}000 = \$500{,}000\,e^{0.06t}$

## Logarithm is an exponent

$\log_{10} 10{,}000 = x$ exponent

base

$10^x = 10{,}000$

$\log = \log_{10}$

$10^4 = 10{,}000$

$\log_{10} 10{,}000 = 4$

Logarithmic form

base

$\log_{\boxed{2}} 64 = x$ exponent

exponential form

base $\rightarrow 2^x = 64$

$2^6 = 64$

$\log_2 64 = \boxed{6}$

$\log_3 81 = x$

$3^x = 81$

$3^4 = 81$

$\log_3 81 = 4$

| Exponential form | logarithmic form |
| --- | --- |
| $\boxed{2}^x = \underline{32}$ | $\log_{\boxed{2}} \underline{32} = x$ $\quad X = 5$ |
| $4^9 = x$ | $\log_4 x = 9$ |