

MTH-PT Trigonometry Session 19 4/4

$$\frac{\cot^2 x}{\cos^2 x \sec^2 x} = \csc^2 x - 1$$

switch → to sin/cos

Look for:

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{\cos^2 x}{\sin^2 x} = \csc^2 x - 1$$

$$\left( \cancel{\cos^2 x} \right) \left( \frac{1}{\cancel{\cos^2 x}} \right)$$

$$\frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

$$\frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}$$

or

$$\begin{array}{r} \cos^2 x = 1 - \sin^2 x \\ + \sin^2 x \quad \quad + \sin^2 x \end{array}$$

$$\boxed{\sin^2 x + \cos^2 x = 1} \quad \checkmark$$

$$\rightarrow \cos^2 x = 1 - \sin^2 x$$

$$1 - \sin^2 x = 1 - \sin^2 x$$

$$\begin{array}{r} \sin^2 x + \cos^2 x = 1 \\ - \sin^2 x \quad \quad - \sin^2 x \\ \hline \cos^2 x = 1 - \sin^2 x \end{array}$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\frac{1 + \cot^2 X}{\sec X} = \left[ \frac{\cot X}{\sin X} \right]$$

$$\frac{1 + \frac{\cos^2 X}{\sin^2 X}}{\frac{1}{\cos X}} = \left[ \frac{\frac{\sin^2 X}{\sin^2 X} + \frac{\cos^2 X}{\sin^2 X}}{\frac{1}{\cos X}} \right]$$

$$\frac{\sin^2 X + \cos^2 X}{\sin^2 X} = \frac{1}{\sin^2 X} \cdot \frac{1}{\cos X}$$

$$\frac{1}{\sin^2 X} \div \frac{1}{\cos X}$$

$$\frac{1}{\sin^2 X} * \frac{\cos X}{1} = \frac{\cos X}{\sin^2 X}$$

$$\frac{\cot X}{\sin X}$$

$$\frac{\cos X}{(\sin X)(\sin X)}$$

$$\frac{\cot X}{\sin X}$$

$$\frac{\sec x + \tan x}{\csc x + 1} = \boxed{\tan x}$$

$$\frac{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}{\frac{1}{\sin x} + 1} = \frac{\frac{1 + \sin x}{\cos x}}{\frac{1}{\sin x} + 1}$$

$$\frac{1 + \sin x}{\cos x}$$

$$\frac{1}{\sin x} + \frac{\sin x}{\sin x}$$

$$\frac{1 + \sin x}{\cos x}$$

$$\frac{1 + \sin x}{\sin x}$$

Keep, change, flip!

$$\frac{1 + \sin x}{\cos x} \div \frac{1 + \sin x}{\sin x}$$

↓      ↓

$$\frac{\cancel{1 + \sin x}}{\cos x} \times \frac{\sin x}{\cancel{1 + \sin x}}$$

→      ↓

$$= \frac{\sin x}{\cos x} = \boxed{\tan x}$$

$$\cos^2 x (1 + \cot^2 x) = \frac{\csc x}{(\sec x)(\tan x)}$$

$$\cos^2 x \left( 1 + \frac{\cos^2 x}{\sin^2 x} \right)$$

$$\cos^2 x \left[ \frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} \right]$$

$$\cos^2 x \left( \frac{\sin^2 x + \cos^2 x}{\sin^2 x} \right)$$

$$\cos^2 x \left( \frac{1}{\sin^2 x} \right) = \frac{\cos^2 x}{\sin^2 x} = \cot^2 x$$

$$\begin{aligned} & \frac{1}{\sin x} \\ & \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} \\ & \frac{1}{\sin x} \\ & \frac{\sin x}{\sin x} \\ & \frac{1}{\sin x} * \frac{\cos^2 x}{\sin x} = \frac{\cos^2 x}{\sin^2 x} \end{aligned}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \text{Sum and Difference Identities}$$

$$\cos \boxed{15^\circ} \quad 30, 45, 60, 90, 120, 135, 150, 180 \dots$$

COS

$$\text{sum: } \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\text{difference } \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos(15) = \cos(45 - 30) \quad A=45^\circ \quad B=30^\circ$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$(\cos 45)(\cos 30) + (\sin 45)(\sin 30)$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$\cos(30) = \boxed{\frac{\sqrt{3}}{2}}$$

$$\cos(90-60) \quad \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

$$\cos(90-60) = (\cos 90)(\cos 60) + (\sin 90)(\sin 60)$$
$$\downarrow \quad \nearrow \quad \downarrow \quad \downarrow$$
$$\cancel{(0)} \left(\frac{1}{2}\right) + (1) \left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\sqrt{3}}{2}}$$

