MTH-PC College Algebra Session 32 1/18

$$\frac{x^2-5x+6}{2x^2+6x}$$

zeros → when numerator goes to zero. $\frac{0}{\ }$

Domain → asymptotes

denominator goes to zero $\frac{\ }{0}$

holes

both numerator and denominator goes to zero $\frac{0}{0}$

End behavior

projection as $x \to \infty$

top heavy → ∞

bottom heavy → 0

great personality → ratio of coefficients

$\underline{-2} * \underline{-3} = 6$

$\underline{-2} + \underline{-3} = -5$

$$\frac{x^2-5x+6}{2x^2+6x} = \frac{(x-2)(x-3)}{2x(x+3)} = \frac{2 \quad 3}{0 \quad -3}$$

$$\frac{2x}{2} = \frac{0}{2}$$

$x = 0$

$x+3=0$

$-3 \quad -3$

$x=-3$

$\frac{5\cancel{x}}{x^{\cancel{2}}}$

$$\frac{\frac{x^2}{x^2} - \frac{5x}{x^2} + \frac{6}{x^2}}{\frac{2x^2}{x^2} + \frac{6x}{x^2}} = \frac{1}{2}$$

zeros: 2, 3

asymptotes $x=0$, $x=-3$

$x \to \infty$

$$\frac{2x^3}{x^2}$$

Top Heavy

$x \to \infty$

$$2\frac{x^3}{x^2} = 2x^{3-2} = 2x \quad 2(\infty) \quad \boxed{\infty}$$

$$\frac{2x^3}{3x^4}$$

Bottom Heavy

$$\frac{2}{3}\frac{x^3}{x^4} = \frac{2}{3}x^{3-4} = \frac{2}{3}x^{-1} = \frac{2}{3x} = \frac{2}{3\infty}$$

$x \to \infty$ $\quad 0$

$$\frac{x^2 - x - 12}{4x + 12}$$

zeros: 4

holes: -3

vertical asymptotes none

end behavior $\boxed{\infty}$ (horizontal)

$$\frac{(x-4)\cancel{(x+3)}}{4\cancel{(x+3)}} \quad x \neq 3$$

$$\text{zeros: } \frac{4 \quad -3}{-3}$$

$$\frac{x^2 - x - 12}{4x+12}$$

Pre-Calculus Chapter 2 Pre-Test

1.) (2.5 pts each, 5 pts total) Determine whether each of the following is a polynomial. If so, identify the degree

a) $f(x) = 2x^5 - 3x^3 + 7x^2 - 9x^1$ yes

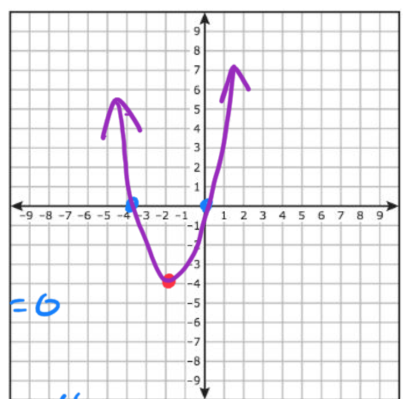
5th degree

b) $f(x) = 5x^3 + 12x^2 + \sqrt{9x}$ No

$\sqrt{9}x^{\frac{1}{2}}$

2.) (5 pts) Graph the quadratic function, which is given in standard form

$f(x) = \boxed{(x + 2)^2 - 4}$



vertex → $(-2,-4)$

zeros

y-int

$(x+2)^2 - 4 = 0$

$+4 \quad +4$

$\sqrt{(x+2)^2} = \sqrt{4}$

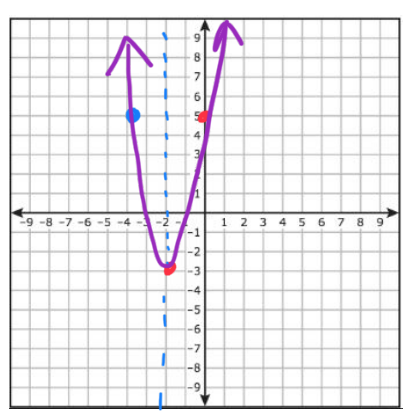
$x+2 = \pm 2$

$-2 \quad -2$

$x = \begin{matrix} 2-2=0 \\ -2-2=-4 \end{matrix}$

3.) (10 pts) Rewrite the quadratic function in standard form by completing the square. Then graph.

$f(x) = 2x^2 + 8x + 5$ ← y-int



$\left(\frac{b}{2}\right)^2$

$\left(\frac{4}{2}\right)^2 = 4$

$\left(\frac{2x^2}{2} + \frac{8x}{2}\right) + 5$

$2(x^2 + 4x \quad) + 5$

$+4 \qquad -4(2)$

$2(x^2 + 4x + 4) - 3$

$2(x+2)^2 - 3$

vertex: $(-2,-3)$

4.) (5 pts) Find all of the real zeros (and their state of multiplicities) for the polynomial.

$$f(x) = 6x^2(x - 2)^4(x + 7)^3$$

$\frac{6x^2}{6} = \frac{0}{6}$

$\sqrt{x^2} = \sqrt{0}$

$x = 0$

zero	0	2	-7
multiplicity	2	4	3

5.) (10 pts) Find a polynomial of minimum degree that has the given zeros.

-2, 0, 1, 3

$X = -2$

$+2 \quad +2$

$X + 2 = 0$

$$\boxed{(x+2)(x)(x-1)(x-3)}$$

6.) (10 pts) For the polynomial function: (a) list each real zero and its multiplicity; (b) determine whether the graph touches or crosses at each x-intercept; (c) find the y-intercept; (d) sketch-ish the graph.

$$f(x) = x^5 - 4x^3$$

factos

$x^3(x^2-4)$

$x^3(x-2)(x+2)$

a) $\sqrt[3]{x^3} = \sqrt[3]{0}$ $\quad$ 0 mult of 3 $\quad$ 2 mult 1 $\quad$ -2 mult 1

$X = 0$

$(x^3)(x-2)(x+2)$

$-6 \quad \ominus \cdot \ominus \quad \ominus = \ominus$

if mult odd → goes thru

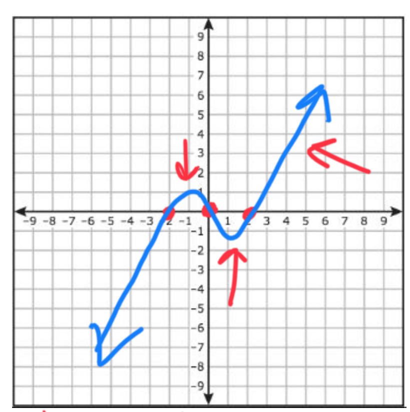
if mult even → bounces

$-6, \; -1, \; 1, \; 6$

$-2 \quad 0 \quad 2$

$\oplus$

$\ominus$



$-1 \quad \ominus \cdot \ominus \cdot \oplus = \oplus$

$1 \quad \oplus \quad \ominus \quad \oplus = -$

$6 \quad \oplus \quad \oplus \quad \oplus = \oplus$

7.) (7.5 pts each, 15 pts total) Divide the polynomials by either long division or synthetic division.

a) $(x^4 - 2x^3 - 7x^2 + 8x + 12) \div (x + 2)$

$x+2 \overline{)\, x^4 - 2x^3 - 7x^2 + 8x + 12}$

x^4

$$\begin{array}{r|rrrrr} -2 & 1 & -2 & -7 & 8 & 12 \\ & \downarrow & -2 & 8 & -2 & -12 \\ \hline & 1 & -4 & 1 & 6 & 0 \end{array}$$

$$\boxed{x^3 - 4x^2 + x + 6}$$

b) $(x^5 + 4x^4 + 3x^2 + 19x + 28) \div (x + 4)$

$0x^3$

$3x + 4 = 0$

$-4 \quad -4$

$\frac{3x}{3} = \frac{-4}{3}$

$x = \frac{-4}{3}$

8.) (10 pts) For the function:

$\downarrow$ $\int q$

$x^4 + 8x^3 + 9x^2 - 38x - 40$

candidate P

a) Find all ~~potential~~ zeros.

$p = 1$ $q = 40$

± 1 ± 2 ± 4 ± 5 ± 8 ± 10 ± 20 ± 40

factors 40

b) Find the number of possible *positive* zeros.

$x^4 + 8x^3 + 9x^2 - 38x - 40$

1

1

c) Find the number of possible *negative* zeros.

$x^4 + 8x^3 + 9x^2 - 38x - 40$

$\downarrow$

$x^4 - 8x^3 + 9x^2 + 38x - 40$

1 2 3

$\boxed{3, 1}$

d) Attempt to ~~find~~ **3 zeros** using long division or synthetic division. Show all work.

attempt

Find 1

9.) (10 pts) Find a polynomial of minimum degree with the following zeros:

-4, 3 - *i*, 3 + *i*

$X = 3 - i \quad X = 3 + i$

$-3+i \quad -3+i \quad -3-i \quad -3-i$

$X - 3 + i = 0 \quad X - 3 - i = 0$

$(X - 3 + i)(X - 3 - i) - (-1)$

$x^2 - 3x - \cancel{ix} - 3x + 9 \cancel{+3i} - i^2$

$\cancel{+ix} \quad \cancel{-3i}$

$x^2 - 3x - 3x + 9 + 1$

$(x^2 - 6x + 10)$

$$\boxed{(x^2 - 6x + 10)(x + 4)}$$

10.) (10 pts) Given a zero of the polynomial, determine all other zeros (real or complex) and write the polynomial as a product of linear factors.

$$x^4 + x^3 - 8x^2 + 4x - 48 \text{ , zero} = 2i$$

$(x+2i)(x-2i) = x^2+4$

$+2i$ assume $-2i$

$x^2+0x+4 \overline{\smash{)}\, x^4+x^3-8x^2+4x-48}$

zeros, holes

11.) (5 pts each, 10 pts total) Find the domain and asymptotes (vertical and horizontal) of each of the following rational functions.

end behavior

a) $\frac{x^2 - 4}{3x^2 - 8x + 4}$

b) $\frac{4x^2 - 3x + 6}{8x^3 - 16x^2 + 8x}$