MTH-PC College Algebra Session 30 1/16

$f(x) = 3x^5 - 4x^4 - x^3 - 52x^2 - 252x - 144$

possible zeros : 5

$3x^5 - 4x^4 - x^3 - 52x^2 - 252x - 144$

$\oplus$ solutions

1

1

change sign of all odd exponents

$3x^5 - 4x^4 - x^3 - 52x^2 - 252x - 144$

$\ominus$ solutions

$-3x^5 - 4x^4 + x^3 - 52x^2 + 252x - 144$

1 2 3 4

4, 2, 0

potential zeros

$3x^5 - 4x^4 - x^3 - 52x^2 - 252x - 144$

"p" "q"

Factor p = 3

±1 $\boxed{\pm 3}$

Factor q

±12 ±6 ±24 ±2 ±72

±36 ±4 ±8 ±18

±3 ±48 ±1 ±144

±9 ±16

$\pm \frac{q}{p}$

$\frac{\pm 12}{\pm 1}$ $\frac{\pm 6}{\pm 1}$ $\frac{\pm 24}{\pm 1}$ $\frac{\pm 2}{\pm 1}$ $\frac{\pm 72}{\pm 1}$

$\frac{\pm 36}{\pm 1}$ $\frac{\pm 4}{\pm 1}$ $\frac{\pm 8}{\pm 1}$ $\frac{\pm 18}{\pm 1}$ $\frac{\pm 3}{\pm 1}$ $\frac{\pm 48}{\pm 1}$ $\frac{\pm 1}{\pm 1}$ $\frac{\pm 144}{\pm 1}$ $\frac{\pm 9}{\pm 1}$

$\frac{\pm 16}{\pm 1}$

$\boxed{\frac{\pm 12}{\pm 1}}$ $\boxed{\frac{\pm 6}{\pm 1}}$ $\boxed{\frac{\pm 24}{\pm 1}}$ $\boxed{\frac{\pm 2}{\pm 1}}$ $\frac{\pm 72}{\pm 1}$ $\boxed{\frac{\pm 16}{\pm 1}}$

$\frac{\pm 36}{\pm 1}$ $\boxed{\frac{\pm 4}{\pm 1}}$ $\boxed{\frac{\pm 8}{\pm 1}}$ $\frac{\pm 18}{\pm 1}$ $\boxed{\frac{\pm 3}{\pm 1}}$ $\boxed{\frac{\pm 48}{\pm 1}}$ $\boxed{\frac{\pm 1}{\pm 1}}$ $\frac{\pm 144}{\pm 1}$ $\frac{\pm 9}{\pm 1}$

40

$\cancel{\frac{\pm 12}{\pm 3}}$ $\cancel{\frac{\pm 6}{\pm 3}}$ $\cancel{\frac{\pm 24}{\pm 3}}$ $\frac{\pm 2}{\pm 3}$ $\cancel{\frac{\pm 72}{\pm 3}}$ $\frac{\pm 16}{\pm 3}$

$\cancel{\frac{\pm 36}{\pm 3}}$ $\frac{\pm 4}{\pm 3}$ $\frac{\pm 8}{\pm 3}$ $\cancel{\frac{\pm 18}{\pm 3}}$ $\cancel{\frac{\pm 3}{\pm 3}}$ $\cancel{\frac{\pm 48}{\pm 3}}$ $\frac{\pm 1}{\pm 3}$ $\cancel{\frac{\pm 144}{\pm 3}}$ $\cancel{\frac{\pm 9}{\pm 3}}$

$x = -1$

$$3x^4 - 7x^3 + 6x^2 - 58x - 194$$

$$x+1 \overline{)\, 3x^5 - 4x^4 - x^3 - 52x^2 - 252x - 144}$$

$-3x^5 \mp 3x^4$

$-7x^4 - x^3$

$+7x^4 + 7x^3$

$6x^3 - 52x^2$

$-6x^3 \mp 6x^2$

$-58x^2 - 252x$

$+58x^2 + 58x$

$-194x - 144$

$+194x + 194$

50

$f(-1) = 50$

$3x^5 - 4x^4 - x^3 - 52x^2 - 252x - 144$

-2

$(x+2)$

$$\begin{array}{c|cccccc} -2 & 3 & -4 & -1 & -52 & -252 & -144 \\ & \downarrow & -6 & 20 & -38 & +180 & +144 \\ \hline & 3 & -10 & 19 & -90 & -72 & 0 \end{array}$$

$f(x) = x^4 - x^3 - 2x^2 - 4x - 24$

poss # of zeros: 4

poss # $\oplus$ solutins $= x^4 - x^3 - 2x^2 - 4x - 24$

1

1

possible # $\ominus$ solutins $= x^4 - x^3 - 2x^2 - 4x - 24$

$\boxed{3, 1}$

$+x^4 + x^3 - 2x^2 + 4x - 24$

1 2 3

possible zeros

$\pm \frac{q}{p}$ $\quad p = 1$ $\quad q = 24$

± 1

$\pm 1 \ \pm 2 \ \pm 3 \ \pm 4$

$\pm 24 \ \pm 12 \ \pm 8 \ \pm 6$

Find a zero

$$\begin{array}{c|ccccc} 1 & 1 & -1 & -2 & -4 & -24 \\ & \downarrow & 1 & 0 & -2 & -6 \\ \hline & 1 & 0 & -2 & -6 & -30 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -1 & -2 & -4 & -24 \\ & \downarrow & 2 & 2 & 0 & -8 \\ \hline & 1 & 1 & 0 & -4 & -32 \end{array}$$

$x^4 - x^3 - 2x^2 - 4x - 24$ Find all zeros

2i is a zero

if 2i is a solution, then -2i is as well

$\pm 2i$ x^2+4

$x = -2i$ $\quad$ $x = 2i$

$+2i \quad +2i$ $\quad$ $-2i \quad -2i$

$i = \sqrt{-1}$

$i^2 = (\sqrt{-1})^2 = -1$

$x + 2i = 0$ $\quad$ $x - 2i = 0$

$(x+2i)(x-2i)$

$x^2 \cancel{-2xi} \cancel{+2xi} - 4i^2$

$-4(-1)$

x^2+4

$x+2$ $\quad$ $(x^2+4)(x^2-x-6)$

$\boxed{(x^2+4)(x-3)(x+2)}$

$$\begin{array}{r} x^2 - x - 6 \\ x^2+0x+4 \enclose{longdiv}{x^4 - x^3 - 2x^2 - 4x - 24} \\ -x^4 \mp 0x^3 \mp 4x^2 \\ \hline -x^3 - 6x^2 - 4x \\ +x^3 + 0x^2 + 4x \\ \hline -6x^2 \quad 0x - 24 \\ +6x^2 + 0x + 24 \\ \hline 0 \end{array}$$

$\frac{-x^3}{x^2}$

$-3 * 2 = -6$

$-3 + 2 = -1$

End Behavior

$$f(x) = \frac{x^3 + x^2 - 2x}{-4x^2 - 12x}$$

$-4x^2 - 12x = 0$

$-4x(x+3) = 0$

hole

$-4x = 0$ $\quad x = 0$

$x + 3 = 0$ $\quad x = -3$

$\;\;-3 \;\; -3$

asymptote

$\{ x^3 + x^2 - 2x$

$x(x^2 + x - 2)$

$x(x+2)(x-1)$

$$x \to \infty \quad \frac{x^3 + x^2 - 2x}{-4x^2 - 12x}$$

Behave like x

$x \to \infty$

Find zeros

Denominator $\frac{\quad}{0}$ undefined

$\boxed{\text{If it cancels, it is a hole}}$

if it does not cancel, asymptote

$$\frac{x^3 + x^2 - 2x}{-4x^2 - 12x}$$

$$\frac{\cancel{x}(x+2)(x-1)}{-4\cancel{x}(x+3)}$$

"TOP HEAVY" ∞

"BOTTOM HEAVY" 0

$$x \to \infty \quad \frac{x^3 - 3x}{x^4 + 8}$$

Bottom Heavy

Behave $\frac{1}{x}$

$\frac{1}{\infty} \to 0$

$$\frac{8\cancel{x^2} \cancel{+ 7}}{2\cancel{x^2} \cancel{+ 10x - 50}} = \frac{8}{2} = 4$$

end behavior