MTH-PC College Algebra Session 29 1/12

$x^4 + x^3 - 10x^2 + 8x$

\# of possible solutions : 4

4th degree quadranomial

$x^4 + x^3 - 10x^2 + 8x \div x - 2$

$(x-2)(x^3+3x^2-4x)$

$$
\begin{array}{r}
x^3 + 3x^2 - 4x \\
x - 2 \,\big)\, x^4 + x^3 - 10x^2 + 8x \\
\underline{-x^4 + 2x^3 } \\
3x^3 - 10x^2 \\
\underline{-3x^3 + 6x^2 } \\
-4x^2 + 8x \\
\underline{+4x^2 \mp 8x} \\
0
\end{array}
$$

$x - 2 = 0$

$+2 \quad +2$

$x = 2$

$$
\begin{array}{r|rrrr}
3 & 12 \\
\end{array}
\quad 4
$$

$(3)(4) = 12$

$$
\begin{array}{c|cccc}
2 & 1 & 1 & -10 & 8 \\
 & \downarrow & 2 & 6 & -8 \\
\hline
 & 1 & 3 & -4 & \boxed{0}
\end{array}
$$

$f(2) = 0$

$f(x) = x^4 + x^3 - 10x^2 + 8x$

$x^3 + 3x^2 - 4x$

zeros : 2, 0, −4, 1

$(x-2)\underbrace{(x^3+3x^2-4x)}$

$(x-2)(x)\underbrace{(x^2+3x-4)}$

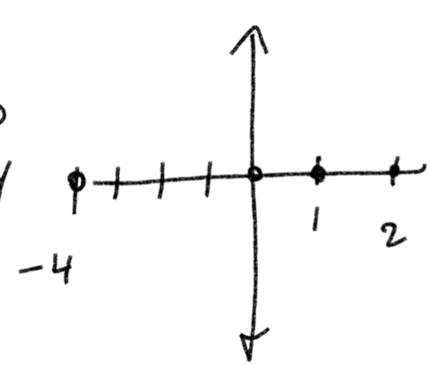
$(x-2)(x)(x+4)(x-1)$

$\uparrow \quad \uparrow$

2 0 −4 1

$x + 4 = 0$

$-4 \quad -4$



$x^4 + x^3 - 10x^2 + 8x \div x + 4$

$$
\begin{array}{r|l}
 & x^3 - 3x^2 + 2x \\
x + 4 & x^4 + x^3 - 10x^2 + 8x \\
 & -x^4 \mp 4x^3 \\
\hline
 & -3x^3 - 10x^2 \\
 & +3x^3 + 12x^2 \\
\hline
 & 2x^2 + 8x \\
 & -2x^2 \mp 8x \\
\hline
 & 0
\end{array}
$$

$f(-4) = 0$

$x + 4 = 0$

$-4 \quad -4$

$x = -4$

$$f(x) = 4x^5 - 3x^4 + 2x^3 - 7x^2 + 9x - 5$$

"p" "q"

# of poss solutions: 5

$\underline{\text{factors } p}$

4

$\underline{1 * 4}$

$2 * 2$

$\underline{\text{factors } q}$

5

1 * 5

$\pm \frac{q}{p}$

$\frac{1}{1} = \pm 1 \quad \pm\frac{1}{4} \quad \pm\frac{1}{2}$

$\frac{5}{1} = \pm 5 \quad \pm\frac{5}{4} \quad \pm\frac{5}{2}$

potential real solutions

insert $+x$ and look for sign changes

$$+\; 4x^5 - 3x^4 + 2x^3 - 7x^2 + 9x - 5$$

$+$ 1 2 3 4 5

5, 3, 1 $+$ solutions

insert $-x$ and look for sign changes

only odd # exponents change

$$4x^5 - 3x^4 + 2x^3 - 7x^2 + 9x - 5$$

$$-4x^5 - 3x^4 - 2x^3 - 7x^2 - 9x - 5$$

$-$ 0

$$x = 1$$

$$4x^4 + x^3 + 3x^2 - 4x + 5$$

$$x - 1 \overline{)\, 4x^5 - 3x^4 + 2x^3 - 7x^2 + 9x - 5}$$

$$-4x^5 + 4x^4$$

$$x^4 + 2x^3$$

$$-x^4 + x^3$$

$$3x^3 - 7x^2$$

$$-3x^3 + 3x^2$$

$$-4x^2 + 9x$$

$$+4x^2 \mp 4x$$

$$5x - 5$$

$$-5x + 5$$

$$0$$

$$\frac{3x^3}{x}$$

$$\frac{5\cancel{x}}{\cancel{x}} = 5$$

$f(x) = 15x^5 + 5x^4 - 81x^3 - 27x^2 + 30x + 10$

Possible number of solution : 5

zero Candidates :

15 "p"	10 "q"
1 * 15	$\boxed{1}$ * $\boxed{10}$
3 * 5	2 * 5
bottom	top

$$\frac{q}{p}$$

$\boxed{1}$ $\pm\frac{1}{1}$ $\pm\frac{1}{15}$ $\pm\frac{1}{3}$ $\pm\frac{1}{5}$

$\boxed{10}$ $\pm\frac{10}{1}$ $\pm\frac{10}{15}$ $\pm\frac{10}{3}$ $\pm\frac{10}{5}$

± 10 $\boxed{\pm\frac{2}{3}}$ $\boxed{\pm 2}$

$\boxed{2}$ $\cancel{\pm\frac{2}{1}}$ $\pm\frac{2}{15}$ $\cancel{\pm\frac{2}{3}}$ $\pm\frac{2}{5}$

$\boxed{5}$ $\pm\frac{5}{1}$ $\cancel{\pm\frac{5}{15}}$ $\pm\frac{5}{3}$ $\cancel{\pm\frac{5}{5}}$

$(+)$ solution

$+15x^5 + 5x^4 - 81x^3 - 27x^2 + 30x + 10$ $\boxed{2, 0}$

$(+)$ 1 2

$(-)$ solution

$-15x^5 + 5x^4 + 81x^3 - 27x^2 - 30x + 10$ $\boxed{3, 1}$

$(-)$ 1 2 3

$x=1$

$$\begin{array}{r} 15x^4+20x^3-61x^2-88x-58 \\ x-1 \overline{)\,15x^5+5x^4-81x^3-27x^2+30x+10} \\ -15x^5+15x^4 \qquad\qquad\qquad\qquad\qquad\quad \\ 20x^4-81x^3 \qquad\qquad\qquad\quad \\ -20x^4+20x^3 \qquad\qquad\qquad\quad \\ -61x^3-27x^2 \qquad\qquad \\ +61x^3 \mp 61x^2 \qquad\qquad \\ -88x^2+30x \qquad \\ +88x^2 \mp 88x \qquad \\ -58x+10 \\ +58x \mp 58 \\ -48 \end{array}$$

$f(1) = -48$

$f(x) = x^4 - x^3 - 5x^2 - x - 6$

poss zeros: 4

i is a zero $\pm i$

$x = i \qquad x = -i$

$-i \; -i \qquad +i \; +i$

$x - i = 0 \qquad x + i = 0$

$(x+i)(x-i)$

$x^2 + \cancel{ix} - \cancel{ix} - i^2$

$x^2 - (\sqrt{-1})^2$

$x^2 - (-1) = x^2+1$

$$\begin{array}{r} x^2-6+\frac{x}{x^2+1} \\ x^2+0x+1 \overline{)\,x^4-x^3-5x^2-x-6} \\ -x^4-0x^3-x^2 \qquad\quad \\ -6x^2-x-6 \\ +6x^2 \; 0x+6 \\ 0 \; -x \; 0 \end{array}$$