

Even function  $f(-x) = f(x)$  Odd Functions

$$f(-3) = f(3)$$

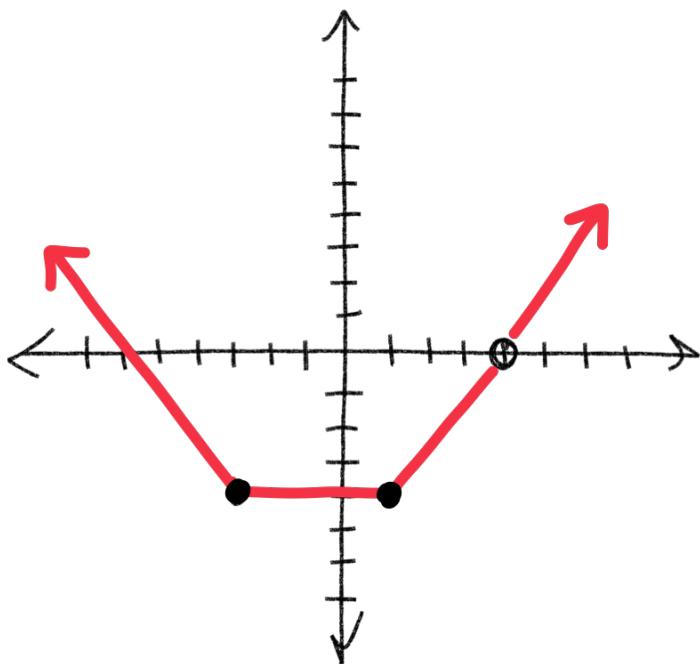
symmetry about y-axis

$$f(-x) = -f(x)$$

$$f(-3) = -f(3)$$

symmetry about  $y=x$ Determine whether the function is even, odd, or mixed  $\rightarrow$  neither.

- even even even  
 ↓ ↓ ↓  
 1.)  $x^8 + x^6 + 6x^6$  shift up  
 even even even  
 ↓ ↓ ↓  
 2.)  $x^7 + x^3 + x^1$   
 odd
- even odd even  
 ↓ ↓ ↓  
 3.)  $x^6 + x^3 + 2$   
 neither
- 4.)  $2x^2 + 3x - 8$   
 neither



$$x \neq 4$$

$$\text{Domain: } (-\infty, 4) \cup (4, \infty)$$

$$\text{Range: } [-4, \infty)$$

$$\text{Increasing: } (1, \infty)$$

$$\text{Decreasing: } (-\infty, -3)$$

$$\text{Constant: } (-3, 1)$$

Average Rate of Change  $\rightarrow$  slope

$$f(x) = 2x$$

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Average Rate of change

From  $f(1)$  to  $f(3)$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

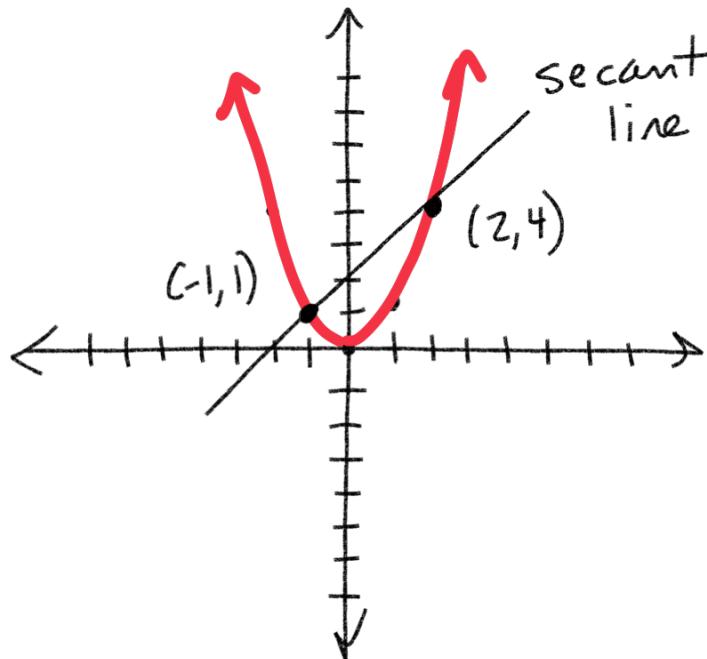
$y \rightarrow f(x)$

$$\frac{f(3) - f(1)}{3 - 1}$$

$$\frac{2(3) - 2(1)}{3 - 1} = \frac{6 - 2}{2} = \frac{4}{2} = \boxed{2}$$

$f(x) = x^2$

Average Rate of Change  
 $f(-1) \rightarrow f(z)$



$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{2 - (-1)}$$

$$\frac{4 - 1}{2 + 1} = \frac{3}{3} = \boxed{1}$$

$$\boxed{\frac{f(2) - f(-1)}{2 - (-1)}}$$

$$\frac{(2)^2 - (-1)^2}{2 - (-1)} = \frac{4 - 1}{3} = \frac{3}{3} = \boxed{1}$$

$$f(x) = x^2 + 4$$

Average Rate of Change

$$x_1 = -2$$

$$\boxed{x_2 = 4}$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

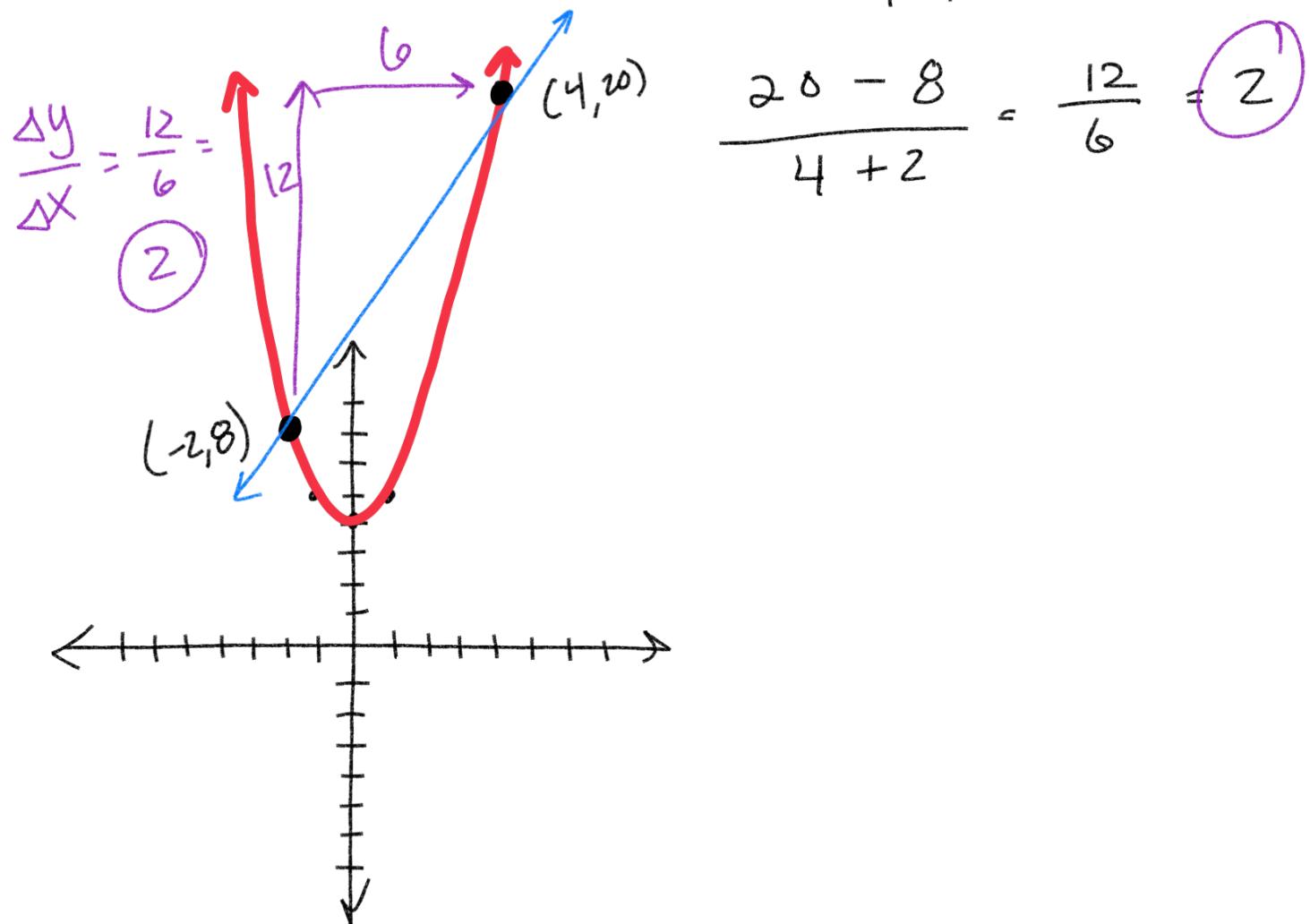
$$\frac{f(4) - f(-2)}{4 - (-2)}$$

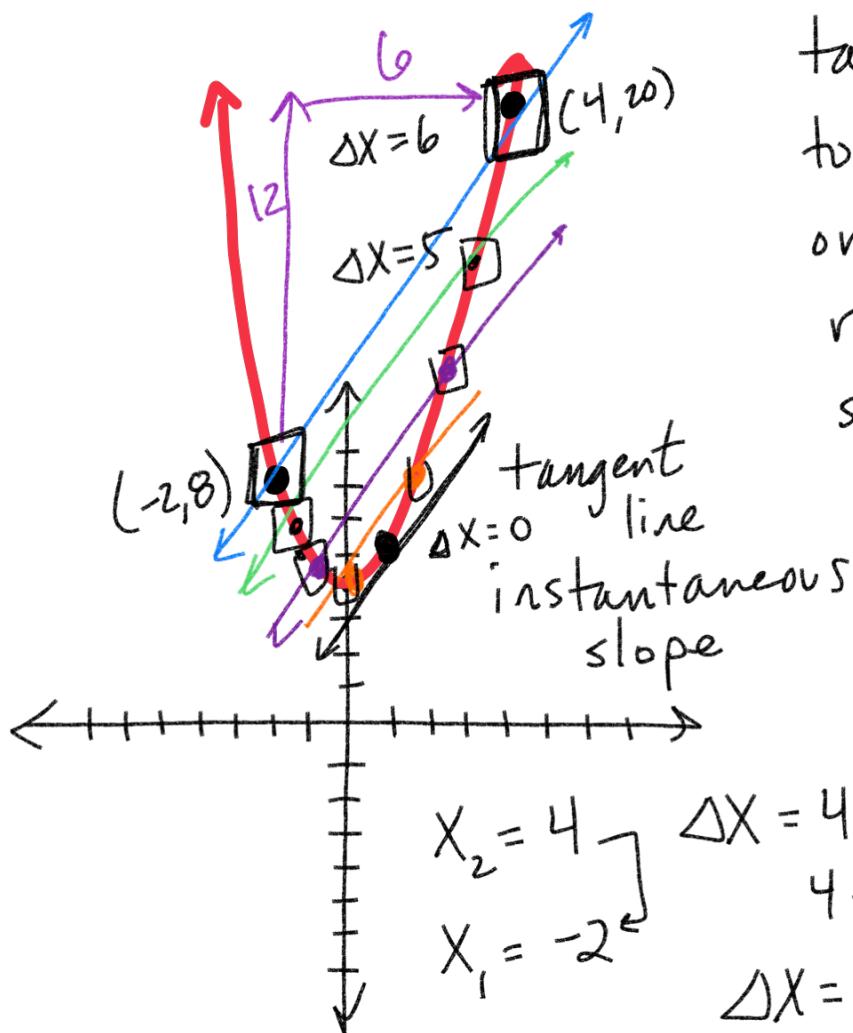
$$f(x) = x^2 + 4$$

$$f(4) = (4)^2 + 4 = 16 + 4 = 20$$

$$f(-2) = (-2)^2 + 4 = 4 + 4 = 8$$

$$\frac{[(4)^2 + 4] - [(-2)^2 + 4]}{4 + 2}$$





tangent line - line that touches the graph in only one place. Its slope represents the instantaneous slope at that point

Average Rate of Change

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$x_2 = 4 \quad \Delta x = 4 - (-2)$$

$$x_1 = -2$$

$$\Delta x = 6$$

$$\frac{f(x_2) - f(x_1)}{-2 - (-2)}$$

$$x_1 = -2$$

$$x_2 = -2 + \Delta x$$

$$-2 + 6$$

$$\cancel{\frac{f(x_2) - f(x_1)}{0}}$$

$$x_2 = x + h$$

$$x_1 = x$$

Difference Quotient

$$\left\{ \begin{array}{l} \cancel{\frac{f(x+h) - f(x)}{x+h - x}} \\ \frac{f(x+h) - f(x)}{h} \end{array} \right.$$

$$f(x) = x^2 + 4$$

$$\frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = (x+h)^2 + 4$$

If  $f(x) = 3x$

$$\frac{3(x+h) - 3x}{h}$$

$$\frac{(x+h)^2 + 4 - (x^2 + 4)}{h}$$

$$\frac{(x+h)(x+h) + 4 - x^2 - 4}{h}$$

$$\frac{x^2 + hx + hx + h^2 + 4 - x^2 - 4}{h}$$

$$x^2 + 4$$

$$2x - 2x$$

$$\frac{hx + hx + h^2}{h} = \frac{2hx + h^2}{h}$$

$$2x + h \quad h=0$$

instantaneous slope =  $2x$

$$f(x) = x^2 - x + 3$$

Average  
Rate  
of  
Change

+  $x+h$   
 Difference Quotient  
 $\frac{(x+h)^2 - (x+h) + 3 - (x^2 - x + 3)}{h}$

$$(x+h)(x+h)$$

$$\cancel{x^2} + 2hx + h^2 \cancel{-x} \cancel{-h} \cancel{+3} \cancel{-x^2} \cancel{+x} \cancel{-3}$$

$$\frac{\cancel{2hx} + h^2 \cancel{-h^{-1}}}{\cancel{h}} = \boxed{2x-1+h}$$

$$\begin{matrix} 2 & \downarrow & 1 & \downarrow & 0 \\ x^2 - x + 3 & \rightarrow & 3x \end{matrix}$$

Derivative  $\boxed{2x-1}$