Algebraic Properties and Proofs

Name ________________________

You have solved algebraic equations for a couple years now, but now it is time to justify the steps you have practiced and now take without thinking… and acting without thinking is a dangerous habit!

The following is a list of the reasons one can give for each algebraic step one may take.

ALGEBRAIC PROPERTIES OF EQUALITY	
ADDITION PROPERTY OF EQUALITY	If $a = b$, then $a + c = b + c$
SUBTRACTION PROPERTY OF EQUALITY	If $a = b$, then $a - c = b - c$
MULTIPLICATION PROPERTY OF EQUALITY	If $a = b$, then $a \cdot c = b \cdot c$
DIVISION PROPERTY OF EQUALITY	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$
DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION or OVER SUBTRACTION	$a(b + c) = ab + ac$ $a(b - c) = ab - ac$
SUBSTITUTION PROPERTY OF EQUALITY	If $a = b$, then b can be substituted for a in any equation or expression
REFLEXIVE PROPERTY OF EQUALITY	For any real number a, $a = a$
SYMMETRIC PROPERTY OF EQUALITY	If $a = b$, then $b = a$
TRANSITIVE PROPERTY OF EQUALITY	If $a = b$ and $b = c$, then $a = c$

Complete the following algebraic proofs using the reasons above. If a step requires simplification by combining like terms, write ***simplify***.

Given: $3x + 12 = 8x - 18$

Prove: $x = 6$

Statements	Reasons
1. $3x + 12 = 8x - 18$	1.
2. $12 = 5x - 18$	2.
3. $30 = 5x$	3.
4. $6 = x$	4.
5. $x = 6$	5.

Given: $3k + 5 = 17$
Prove: $k = 4$

Statements	Reasons
1. $3k + 5 = 17$	1.
2. $3k = 12$	2.
3. $k = 4$	3.

Given: $-6a - 5 = -95$
Prove: $a = 15$

Statements	Reasons

Given: $3(5x + 1) = 13x + 5$
Prove: $x = 1$

Statements	Reasons

Given: $7y - 84 = 2y + 61$

Prove: $y = 29$

Statements	Reasons

Given: $4(5n + 7) - 3n = 3(4n - 9)$

Prove: $n = -11$

Statements	Reasons

Given: $4.7 \quad 2f - 0.5 \quad = -6 \quad 1.6f - 8.3f$

Prove: $y = -\dfrac{47}{616}$

Statements	Reasons

Geometric Properties

We have discussed the RST (Reflexive, Symmetric, and Transitive) properties of equality. We could prove that these also apply for congruence… but we won't. We are just going to accept it…
I know, you're disappointed.

PROPERTIES OF CONGRUENCE	
REFLEXIVE PROPERTY OF CONGRUENCE	For any geometric figure *A*, $A \cong A$.
SYMMETRIC PROPERTY OF CONGRUENCE	If $A \cong B$, then $B \cong A$.
TRANSITIVE PROPERTY OF CONGRUENCE	If $A \cong B$ and $B \cong C$, then $A \cong C$
Additional Reasons for Proofs	
DEFINITIONS	
POSTULATES	
PREVIOUSLY PROVED THEOREMS	
ALGEBRAIC PROPERTIES	

Elementary Geometric Proofs

Using Definitions

Given: $\overline{XY} \cong \overline{BC}$
Prove: $XY = BC$

Statements	Reasons

Given: $\angle A \cong \angle Z$
Prove: $m\angle A = m\angle Z$

Statements	Reasons

Using the Transitive Property and Substitution

Given: $m\angle 1 = 45^\circ$; $m\angle 2 = m\angle 1$

Prove: $m\angle 2 = 45^\circ$

Statements	Reasons

You should be aware that there are many ways to complete a proof. In fact, the following website has 79 distinct proofs for the most famous of all theorems, the Pythagorean Theorem.

http://www.cut-the-knot.org/pythagoras/index.shtml

Even the simple proof above could be done in at least two ways. The last statement could have been justified using SUBSTITUTION or the TRANSITIVE PROPERTY. These properties are similar, but no the same:

SUBSTITUTION works only on NUMBERS (=), while the TRANSITIVE PROPERTY can be used to describe relationships between FIGURES or NUMBERS (= or ≅). Keep this in mind.

Given: $\angle 1 \cong \angle 2$; $\angle 1 \cong \angle 3$

Prove: $\angle 2 \cong \angle 3$

Statements	Reasons

Using Multiple Reasons

Given: $m\angle A = 90^\circ$; $\angle A \cong \angle Z$
Prove: $\angle Z$ is a right angle

Statements	Reasons

Given: $m\angle 1 = 90^\circ$; $\angle 1 \cong \angle 2$; $\angle 2 \cong \angle 3$
Prove: $\angle 3$ is a right angle

Statements	Reasons

Given: $m\angle O = 180^\circ$; $m\angle P = m\angle S$; $\angle O \cong \angle P$
Prove: $\angle S$ is a straight angle

Statements	Reasons

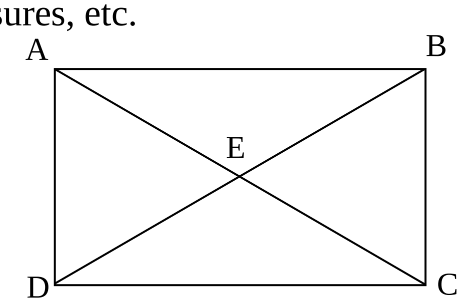
DEFINITIONS AND POSTULATES REGARDING SEGMENTS	
SEGMENT ADDITION POSTULATE	If *C* is between *A* and *B*, then AC + CB = AB
DEFINITION OF SEGMENT CONGRUENCE	If $\overline{AB} \cong \overline{CD}$, then AB = CD
DEFINITION OF A SEGMENT BISECTOR	*A geometric figure that divides a segment in to two congruent halves*
DEFINITION OF A MIDPOINT	*A point that bisects a segment*
DEFINITIONS AND POSTULATES REGARDING ANGLES	
ANGLE ADDITION POSTULATE	If *C* is on the interior of $\angle ABD$, then $m\angle ABC + m\angle CBD = m\angle ABD$
DEFINITION OF ANGLE CONGRUENCE	If $\angle A \cong \angle B$, then $m\angle A = m\angle B$
DEFINITION OF AN ANGLE BISECTOR	*A geometric figure that divides a angle in to two congruent halves*

Proofs with Pictures

It is often much easier to plan and finish a proof if there is a visual aid. Use the picture to help you plan and finish the proof. Be sure that as you write each statement, you make the picture match your proof by inserting marks, measures, etc.

Given: E is the midpoint of $\overline{AC}$ and $\overline{BD}$; $\overline{ED} \cong \overline{EC}$

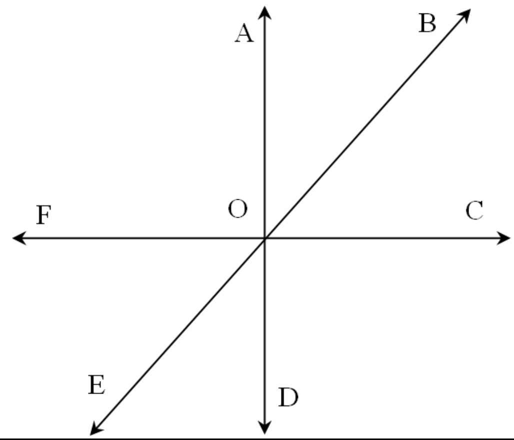
Prove: $\overline{AE} \cong \overline{BE}$



Statements	Reasons

Given: $\overrightarrow{OB}$ bisects $\angle AOC$;
$\overrightarrow{OE}$ bisects $\angle DOF$;
$\angle AOB \cong \angle DOE$

Prove: $\angle EOF \cong \angle BOC$

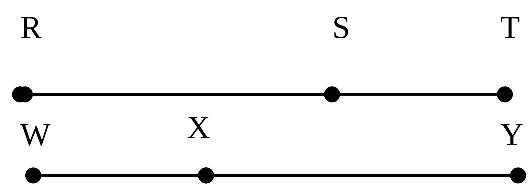


Statements	Reasons

Elementary Geometric Proofs

Segments

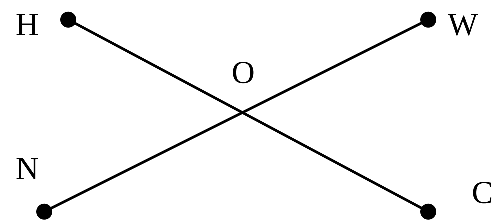
Given: $\overline{RT} \cong \overline{WY}$; $\overline{ST} \cong \overline{WX}$
Prove: $\overline{RS} \cong \overline{XY}$



Statements	Reasons

Given: O is the midpoint of $\overline{NW}$; $\overline{NO} \cong \overline{OC}$

Prove: $\overline{OC} \cong \overline{OW}$



Statements	Reasons
1. O is the midpoint of $\overline{NW}$	1.
2. $\overline{NO} \cong \overline{OW}$	2.
3.	3. Given
4. $\overline{OC} \cong \overline{OW}$	4.

Given: $\overline{EF} \cong \overline{GH}$

Prove: $\overline{EG} \cong \overline{FH}$

Statements	Reasons
1. $\overline{EF} \cong \overline{GH}$	1.
2. EF = GH	2.
3. EF + FG = GH + FG	3.
4. EF + FG = EG; GH + FG = FH	4.
5. EG = FH	5.
6. $\overline{EG} \cong \overline{FH}$	6.

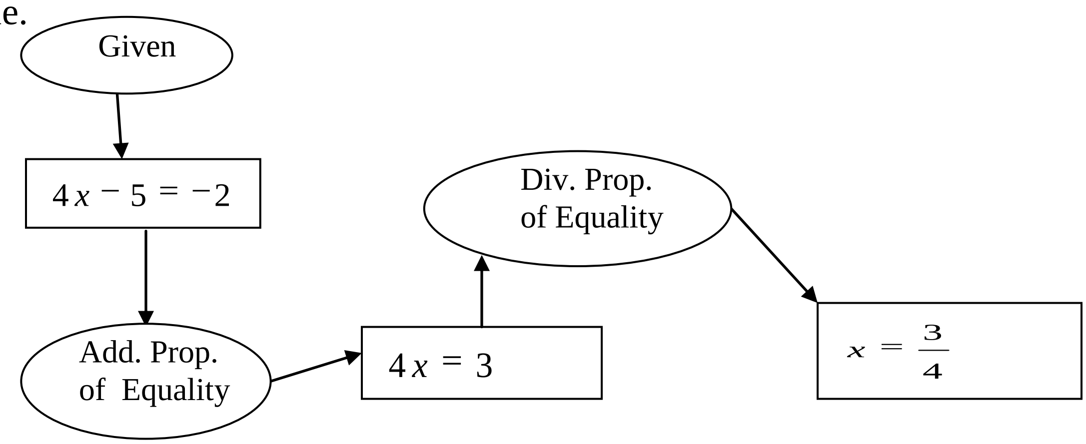
Flow Proofs

Proofs do not always come in two-column format. Sometimes they are more visual, as you will see in this example.

Flow Proof

Given: $4x - 5 = -2$

Prove: $x = \frac{3}{4}$



Complete the flow chart for the following proof.

Given: $AC = CE; AB = DE$

Prove: C is the midpoint of $\overline{BD}$