Calculus & Trigonometry

Chapter 6 Pre-Test

1.) (20 pts total, 5 pts each) Simplify each of the following trigonometric expressions.

a) $(sin^2 x)(cot^2 x + 1)$

$\cancel{\sin^2 x}\left(\frac{\cos^2 x}{\cancel{\sin^2 x}} + 1\right) = \cos^2 x + \sin^2 x = \boxed{1}$

b) $(sin\, x - cos\, x)(sin\, x + cos\, x)$

c) $\frac{1 - cos^4 x}{1 + cos^2 x}$

$\frac{\cancel{(1 + \cos^2 x)}(1 - \cos^2 x)}{\cancel{1 + \cos^2 x}}$

$1 - \cos^2 x = \boxed{\sin^2 x}$

$\sin^2 x + \cos^2 x = 1$

$-\cos^2 x \quad -\cos^2 x$

$\sin^2 x = 1 - \cos^2 x$

d) $1 - \frac{sin^2 x}{1 - cos\, x}$

2.) (40 pts total, 5 pts each) Verify each of the following trigonometric identities.

a) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = \cancel{\cos^2 x}$ 2

$(\sin x + \cos x)(\sin x + \cos x)$

$\sin^2 x + \cos x \sin x + \cos x \sin x + \cos^2 x$

$\sin^2 x + \cancel{2\cos x \sin x} + \cos^2 x = 1$

$\sin^2 x + \cos^2 x - \cancel{2\cos x \sin x}$ 1 $\quad 1+1=2$

$(\sin x - \cos x)(\sin x - \cos x)$

b) $\tan x + \cot x = \csc x \sec x$

c) $\frac{2 - \sin^2 x}{\cos x} = \sec x + \cos x$

$\frac{2 - \sin^2 x}{\cos x} = \frac{1}{\cos x} + \frac{\cos^2 x}{\cos x}$

$2 - \sin^2 x = 1 + \cos^2 x$

$\sin^2 x + \cos^2 x = 1$

$-\cos^2 x \quad -\cos^2 x$

$\sin^2 x = 1 - \cos^2 x$

$2 - (1 - \cos^2 x)$

$2 - 1 + \cos^2 x$

$\boxed{1 + \cos^2 x}$

d) $\frac{1}{csc^2 x} + \frac{1}{sec^2 x} = 1$

e) $\frac{1}{1 - sin\, x}\frac{(1+\sin x)}{(1+\sin x)} + \frac{1}{1 + sin\, x}\frac{(1-\sin x)}{(1-\sin x)} = 2sec^2 x$

$\frac{1+\sin x}{1-\sin^2 x}$ $\frac{1-\sin x}{1-\sin^2 x} = \frac{2}{\cos^2 x}$

$\frac{\cancel{1+\sin x} + \cancel{1-\sin x}}{\cos^2 x} = \frac{2}{\cos^2 x}$

f) $\frac{sin^2 x}{1 - cos\, x} = 1 + cos\, x$

g) $\sec x + \tan x = \frac{1}{\sec x - \tan x}$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x} = \frac{1}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}$$

$$\frac{1+\sin x}{\cos x} \qquad \frac{1}{\frac{1-\sin x}{\cos x}}$$

$$\frac{1+\sin x}{\cos x} = \frac{\cos x}{1-\sin x}$$

$$(1+\sin x)(1-\sin x) = \cos^2 x$$

$$1-\sin^2 x = \cos^2 x \checkmark$$

$$\cos^2 x = \cos^2 x$$

h) $\frac{\cos^2 x + 1 + \sin x}{\cos^2 x + 3} = \frac{1 + \sin x}{2 + \sin x}$

$$\frac{(\cos x)(1+\sin x)}{(1-\sin x)(1+\sin x)}$$

$$\frac{\cancel{\cos x} + \cancel{\cos x}\sin x}{\cos^{\cancel{2}} x}$$

$$\frac{1+\sin x}{\cos x}$$

3.) (10 pts total, 5 pts each) Write each expression as a single trigonometric function.

a) $\sin(4x)\sin(3x) + \cos(4x)\cos(3x)$

b) $sin\ x\ cos\ (8x)\ -\ cos\ x\ sin\ (8x)$

4.) (10 pts total, 5 pts each) Use double angle identities to solve each of the following.

a) $If\ cos\ x\ =\ \frac{5}{24}\ and\ sin\ x\ <\ 0,\ find\ tan\ (2x)$

b) $sin\ 15°\ cos\ 15°$

5.) (10 pts total, 2 pts each) Find the exact values of each problem

a) $cos\ 15°$

b) $tan\ 202.5°$

6.) (5 pts total) Write the product as a sum or difference.

$$4sin\ (3x)\ sin\ (4x)$$

7.) (5 pts total) Write the expression as a product.

$$cos\,(8x) \;+\; cos\,(3x)$$