M-CT Calc & Trig 2/28

Double angle identity

$$\sin(2A) = 2\sin A\cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$1 - 2\sin^2 A$$

$$2\cos^2 A - 1$$

$$\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$$

Half angle identity

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\pm\sqrt{1-\cos A}}{\sqrt{1+\cos A}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin\left(\frac{A}{2}\right)}{\cos\left(\frac{A}{2}\right)}$$

Product-to-sum identities

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$4\cos(3x)\cos(2x)$$

$A = 3x \quad B = 2x$

$$4\cos(3x)\cos(2x) = 4 \cdot \frac{1}{2}\left[\cos(3x+2x) + \cos(3x-2x)\right]$$

$$4 \cdot \frac{1}{2}\left[\cos(5x) + \cos x\right]$$

$$2\cos(5x)\cos x$$

$A = 60^\circ \quad B = 30^\circ$

$$\cos 60^\circ \cos 30^\circ = \cos 30^\circ \cos 60$$

$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} \qquad \frac{\sqrt{3}}{2} \quad \frac{1}{2} = \frac{\sqrt{3}}{4}$$

$$(\cos 60^\circ)(\cos 30^\circ) = \frac{1}{2}\left[\cos(60+30) + \cos(60-30)\right]$$

$$\frac{1}{2}\left[\cos 90 + \cos 30\right]$$

$$\frac{1}{2}\left[0 + \frac{\sqrt{3}}{2}\right]$$

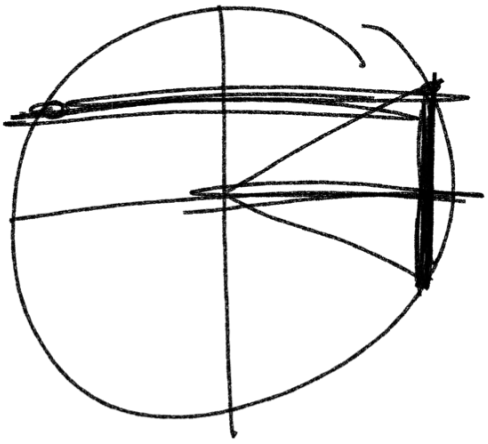
$$= \frac{\sqrt{3}}{4}$$

$$(\cos 30)(\cos 60) = \frac{1}{2}\left[\cos(30+60) + \cos(30-60)\right]$$

$$\cos 90 + \cos(-30)$$

$$\frac{1}{2}\left[\frac{\sqrt{3}}{2}\right]$$

$$\frac{\sqrt{3}}{4}$$



$$\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right]$$

1.) $-3\sin(2x)\sin(4x)$

$$-3 * \frac{1}{2}\left[\cos(2x-4x) - \cos(2x+4x)\right)$$

$$\boxed{-\frac{3}{2}\cos(-2x) - \cos(6x)}$$

2.) $\cos(10x)\sin(5x)$

$$\frac{1}{2}\left[\sin(5x+10x) + \sin(5x-10x)\right]$$

$$\boxed{\frac{1}{2}\left(\sin 15x + \sin(-5x)\right)}$$

Sum-to-Product

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$A=30 \quad B=60$

$$\sin 30 + \sin 60 \qquad 2\sin\left(\frac{90}{2}\right)\cos\left(\frac{30-60}{2}\right)$$

$$\frac{1}{2} \qquad \frac{\sqrt{3}}{2} \qquad\qquad 2\sin 45^\circ \cos -15^\circ$$

$$\boxed{\frac{1+\sqrt{3}}{2}} \qquad \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right)\cos(-15)$$

$$\cos(-15^\circ) = \cos(45-60)$$

$$\cos(A-B) \quad \cos A\cos B + \sin A \sin B$$

$$\cos 45 \cos 60 + \sin 45 \sin 60$$

$$\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \qquad \frac{\sqrt{2}+\sqrt{6}}{4}$$

$$\left(\sqrt{2}\right)\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right) = \frac{\sqrt{4}+\sqrt{12}}{4} = \frac{\cancel{2}+\cancel{2}\sqrt{3}}{\cancel{4}_{2}}$$

$$\boxed{\frac{1+\sqrt{3}}{2}}$$

$$\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

MEMORIZE ALL
FOR THE TEST!!
NO SHEET!!!

$$\cos(5x) + \cos(3x) = 2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)$$

$$2\cos\frac{8x}{2}\cos\frac{2x}{2}$$

$$\boxed{2\cos 4x \cos x}$$

$y = 3x$

Find the inverse

$\frac{x}{3} = \frac{3y}{3}$

$y = \frac{x}{3}$

$y = 3x$

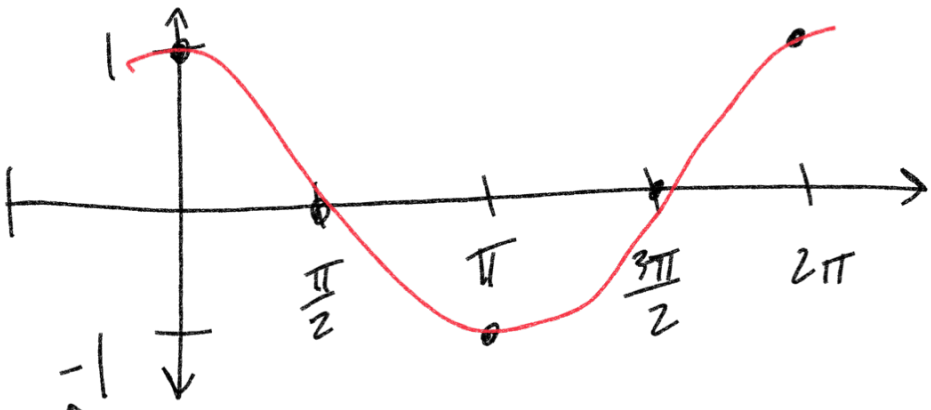
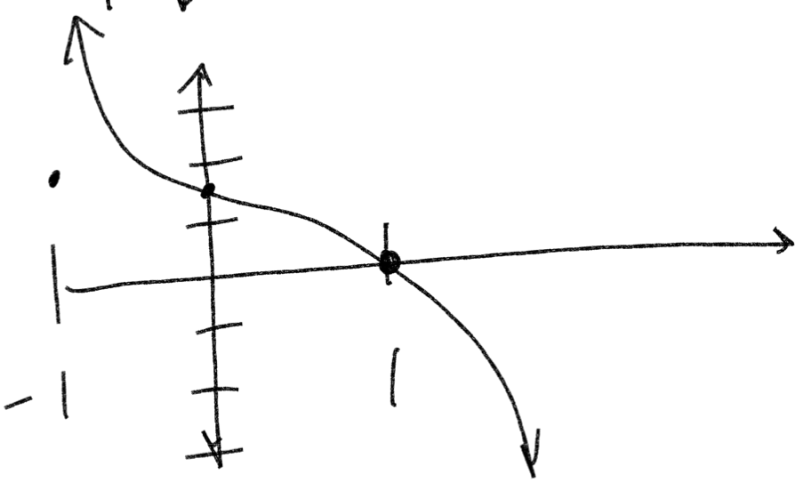
x	y
0	0
1	3
2	6
3	9

Inverse

$y = \frac{x}{3}$

x	y
0	0
3	1
6	2
9	3

cos x



x	y
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1