

Reteaching 4-1

Ratio and Proportion

OBJECTIVE: Solving proportions**MATERIALS:** None

An equation that states that two ratios are equal is called a proportion. In a proportion, the cross products are equal.

Example

Use cross products to find out if the proportion $\frac{2}{7} = \frac{10}{40}$ is true.

$$\frac{2}{7} = \frac{10}{40}$$

$$2 \cdot 40 = 7 \cdot 10 \quad \leftarrow \text{Write cross products.}$$

$$80 = 70 \quad \leftarrow \text{Simplify.}$$

$$80 \neq 70 \quad \leftarrow \text{Proportion is not true since 80 does not equal 70.}$$

Use cross products to write and solve equations involving proportions.

Solve: $\frac{5}{6} = \frac{25}{x}$

$$\frac{5}{6} = \frac{25}{x} \quad \leftarrow \text{The cross products are } 5x \text{ and } 6 \cdot 25 \text{ or } 150.$$

$$5x = 6 \cdot 25 \quad \leftarrow \text{Set cross products equal to each other.}$$

$$5x = 150 \quad \leftarrow \text{Simplify.}$$

$$\frac{5x}{5} = \frac{150}{5} \quad \leftarrow \text{Use the Division Property of Equality.}$$

$$x = 30 \quad \leftarrow \text{Simplify.}$$

Exercises

Determine if the proportions are true. (Hint: the cross products should be equal.)

1. $\frac{6}{10} = \frac{12}{20}$

2. $\frac{4}{5} = \frac{7}{8}$

3. $\frac{33}{22} = \frac{24}{16}$

Solve each proportion.

4. $\frac{x}{5} = \frac{2}{10}$

5. $\frac{9}{180} = \frac{n}{60}$

6. $\frac{2}{x} = \frac{8}{36}$

7. $\frac{2}{6} = \frac{4}{x}$

8. $\frac{30}{125} = \frac{n}{100}$

9. $\frac{3}{18} = \frac{t}{6}$

10. $\frac{t}{5} = \frac{3}{5}$

11. $\frac{28}{8} = \frac{7}{x}$

12. $\frac{9}{n} = \frac{18}{2}$

Practice 4-1**Ratio and Proportion****Find each unit rate.**

1. \$60 for 8 h 2. $\frac{\$3}{4 \text{ lb}}$ 3. $\frac{861 \text{ bagels}}{3 \text{ d}}$ 4. $\frac{850 \text{ cal}}{1.25 \text{ h}}$
5. An 8-ounce bottle of lotion costs \$4.50. What is the cost per ounce?
6. A pound of coffee costs \$14.99. What is the cost per ounce?

Which pairs of ratios could form a proportion? Justify your answer.

7. $\frac{10}{24}, \frac{7}{18}$ 8. $\frac{6}{9}, \frac{10}{15}$ 9. $\frac{3}{4}, \frac{18}{24}$ 10. $\frac{16}{2}, \frac{8}{1}$ 11. $-\frac{4.8}{4}, -\frac{6.4}{5}$

Solve each proportion.

12. $\frac{g}{5} = \frac{6}{10}$ 13. $\frac{z}{4} = \frac{7}{8}$ 14. $\frac{13.2}{6} = \frac{m}{12}$ 15. $-\frac{m}{5} = -\frac{2}{5}$
16. $\frac{5.5}{11} = \frac{x}{5}$ 17. $-\frac{2}{3} = -\frac{10}{t}$ 18. $\frac{4}{6} = \frac{x}{24}$ 19. $\frac{s}{3} = \frac{7}{10}$
20. $\frac{4}{9} = \frac{10}{r}$ 21. $\frac{x}{4.8} = \frac{6}{3.2}$ 22. $\frac{5}{4} = \frac{c}{12}$ 23. $-\frac{32}{h} = -\frac{1}{3}$
24. $\frac{2}{6} = \frac{p}{9}$ 25. $\frac{f}{6} = \frac{3}{4}$ 26. $\frac{15}{a} = \frac{3}{8}$ 27. $\frac{3}{4} = \frac{k}{24}$
28. $\frac{a}{6} = \frac{3}{9}$ 29. $\frac{4}{5} = \frac{k}{9}$ 30. $\frac{3}{y} = \frac{5}{8}$ 31. $\frac{t}{7} = \frac{9}{21}$
32. $\frac{2}{9} = \frac{10}{x}$ 33. $\frac{x}{15} = \frac{3}{4}$ 34. $\frac{18}{11} = \frac{49.5}{x}$ 35. $\frac{2}{1.2} = \frac{5}{x}$
36. $-\frac{x-1}{4} = \frac{2}{3}$ 37. $\frac{3}{6} = \frac{x-3}{8}$
38. $\frac{2x-2}{14} = \frac{2x-4}{6}$ 39. $\frac{x+2}{x-2} = \frac{4}{8}$
40. $\frac{x+2}{6} = \frac{x-1}{12}$ 41. $-\frac{x+8}{10} = -\frac{x-3}{2}$
42. You are riding your bicycle. It takes you 28 min to go 8 mi. If you continue traveling at the same rate, how long will it take you to go 15 mi?
43. Suppose you traveled 84 mi in 1.5 h. Moving at the same speed, how many mi would you cover in $3\frac{1}{4}$ h?
44. A canary's heart beats 130 times in 12 s. Use a proportion to find how many times its heart beats in 50 s.
45. Your car averages 18 mi per gal on the highway. If gas costs \$1.85 per gal, how much does it cost in dollars per mi to drive your car on the highway?

Reteaching 4-2

Proportions and Similar Figures

OBJECTIVE: Finding missing measures of similar figures

MATERIALS: None

Setting up a proportion can help determine the missing lengths from similar figures. Remember the following:

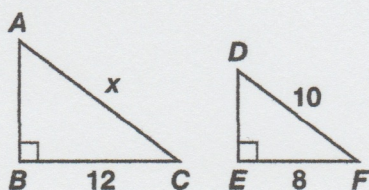
- Always compare corresponding sides when writing the ratios.
- Be consistent by keeping sides of the same figure either in the denominators or numerators.

Example

$\triangle ABC \sim \triangle DEF$

Find the length of x .

BC corresponds to EF
 AC corresponds to DF



$$\frac{AC}{DF} = \frac{BC}{EF}$$

← Notice the numerators, AC and BC, are sides of the same triangle.

$$\frac{x}{10} = \frac{12}{8}$$

← Substitute appropriate values.

$$8x = 10 \cdot 12$$

← Write the cross products.

$$\frac{8x}{8} = \frac{120}{8}$$

← Divide each side by 8.

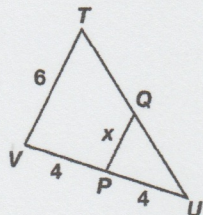
$$x = 15$$

← Simplify.

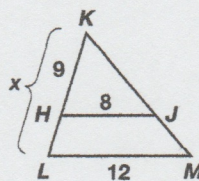
Exercises

Find the length of x .

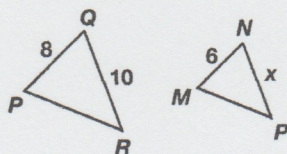
1. $\triangle TUV \sim \triangle QUP$



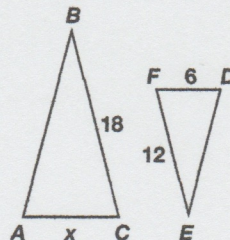
2. $\triangle KLM \sim \triangle KHJ$



3. $\triangle PQR \sim \triangle MNP$



4. $\triangle ABC \sim \triangle DEF$

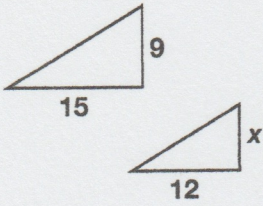


Practice 4-2

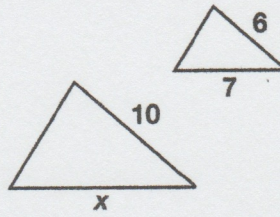
Proportions and Similar Figures

Each pair of figures is similar. Find the length of x .

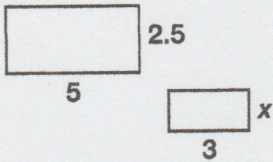
1.



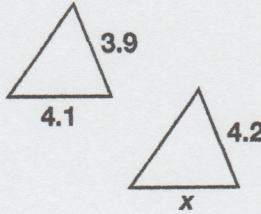
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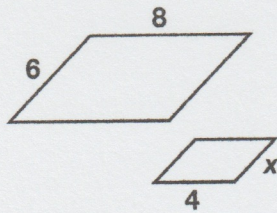
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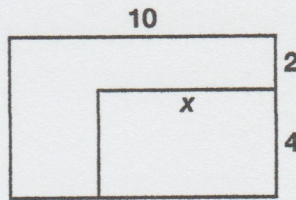
4.



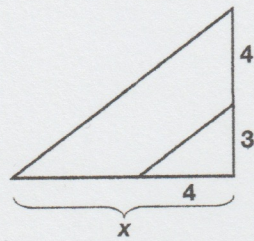
5.



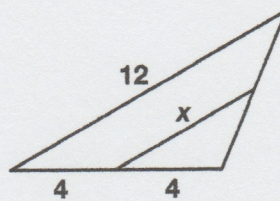
6.



7.



8.



Use a proportion to solve.

9. $\triangle ABC$ is similar to $\triangle XYZ$. The length AB is 10. The length BC is 7. Find the length XY if the length YZ is 14.
10. Marty has a scale model of a car. The scale is 1 in. : 32 in. If the model is 6.75 in. long, how long is the actual car?
11. A blueprint scale is 1 in. : 12 ft. The width of a building is 48 ft. What is the width of the building on the blueprint?
12. Angie is using similar triangles to find the height of a tree. A stick that is 5 ft tall casts a shadow that is 4 ft long. The tree casts a shadow that is 22 ft long. How tall is the tree?
13. $\triangle ABC$ is similar to $\triangle XYZ$. The length AC is 10. The length BC is 16. What is the length XZ if the length YZ is 12?
14. A map has a scale of 1 in. : 25 mi. Two cities are 175 mi apart. How far apart are they on the map?

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Reteaching 4-3

Proportions and Percent Equations

OBJECTIVE: Using equations to solve problems involving percents

MATERIALS: None

To solve percent problems, you can often represent words with math symbols. This table shows some words and the symbols you can use to represent them.

word	what	is	of
symbol	n (or another variable)	=	\times

Example

What is 30% of 20?

What is 30% of 20?

← Draw a triangle around the word that represents the variable. Draw a rectangle around the word that represents =. Circle the word that represents \times .

What is 30% of 20?

← Copy just the shapes and numbers, including percent.

n = 0.30 \times 20

← Inside each shape, represent the word with the correct symbol. Rewrite percents as decimals.

n = 0.30 \times 20

← Write the equation.

n = 6

← Solve the equation.

30% of 20 is 6.

← Answer the question.

Exercises

Use rectangles, circles, and triangles to rewrite and answer each question.

- What is 10% of 50?
- 8 is 40% of what?
- 25 is what percent of 100?
- 7 is what percent of 35?
- What is 30% of 50?
- 3 is 25% of what?
- 0.4 is what percent of 0.8?
- What is 25% of 25?
- What is 25% of $\frac{4}{7}$?
- Kamala is 15 years old and her brother is 20. Kamala's age is what percent of her brother's?
- Luis worked 12 h at the school library. That represents 25% of the total hours he has volunteered to work. How many hours has he volunteered to work?
- The sales tax rate is 6%. What is the sales tax on \$55?

Practice 4-3**Proportions and Percent Equations****Solve each problem.**

- | | | |
|----------------------------------|-------------------------------|------------------------------|
| 1. 25% of what is 28? | 2. What percent of 72 is 18? | 3. 60% of what is 45? |
| 4. What percent of 12 is 6? | 5. What is 60% of 12? | 6. 75% of what is 48? |
| 7. What is 20% of 650? | 8. What percent of 150 is 90? | 9. What percent of 90 is 63? |
| 10. What is 38% of 60? | 11. 22.5% of what is 42? | 12. 45% of what is 99? |
| 13. What percent of 210 is 10.5? | 14. 160% of what is 124? | 15. What is 39% of 1500? |
| 16. What is 250% of 14? | 17. What percent of 20 is 36? | 18. What is 8.25% of 160? |

Write an equation to model each question and solve.

19. Pablo has a goal to lose 25 lb. He has lost 16 lb. What percent of his goal has he reached?
20. You spent 16% of your vacation money on food. If you spent \$48 on food, how much money did you spend on your vacation?
21. A writer earns \$3400 a month. Last month she spent \$204 on food. What percent of her income was spent on food?
22. Kiko spends 30% of her monthly income on rent. If she pays \$810 for rent each month, what is her monthly income?
23. Suppose that 62.5% of freshmen entering a college graduate from it. If there are 2680 freshmen, how many will graduate from that college?

The formula for determining simple interest is $I = prt$. Using this formula, solve the following problems.

24. You invest \$1500 for three years. Find the amount of simple interest you earn at an annual rate of 8.25%.
25. Suppose you invested \$1200 for four years. You earned \$312 in simple interest. What is the interest rate?
26. Suppose you invested some money at 8% simple interest for five years. If you received \$500 in interest, how much money did you invest?

Write an equation to model each question and solve.

- | | | |
|-------------------------|-------------------------------|------------------------------|
| 27. What is 7% of 480? | 28. What percent of 80 is 48? | 29. 90% of what is 27? |
| 30. What is 150% of 26? | 31. 125% of what is 175? | 32. What is 10.25% of 280? |
| 33. What is 35% of 360? | 34. What percent of 36 is 9? | 35. 75% of what is 90? |
| 36. 45% of what is 36? | 37. What is 80% of 120? | 38. What percent of 20 is 8? |
| 39. 25% of what is 92? | 40. What percent of 30 is 90? | 41. What is 39% of 800? |

Reteaching 4-4

Percent of Change

OBJECTIVE: Finding percent of change

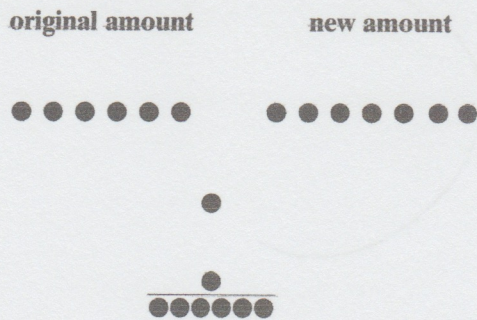
MATERIALS: About 35 counters such as beans or paper clips

Use this ratio to find the percent of change from an original amount to a new amount:

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}}$$

Example

Jana's pay changed from \$6/h to \$7/h. Find the percent of change in her pay and whether it is a percent of increase or of decrease.



$$\frac{\text{amount of change}}{\text{original amount}} = \frac{1}{6}$$

The percent of increase is 16.67%.

More counters under the original amount represents a percent of decrease.
More counters under the new amount represents a percent of increase.

- ← Divide your paper into two areas, labeled "original amount" on the left and "new amount" on the right.
- ← Place 6 counters to represent \$6 under the original amount and 7 counters under the new amount.
- ← Find the difference between the amount of counters on the left and on the right.
- ← Put the counters representing the difference (amount of change) over the counters representing the original amount.
- ← Write the ratio represented by the counters.
- ← Write the ratio as a percent.

Exercises

Use counters to find each percent of change. Describe the percent as an increase or decrease. Round percents to the nearest integer.

1. 8 in. to 12 in.
2. 7 min to 5 min
3. \$3 to \$4
4. 2 lb to 4 lb

Find each percent of change. Describe the percent as an increase or decrease. Round percents to the nearest integer.

5. Today eight students leave your classroom.
6. You put 5 pencils in a box that already has 12 pencils.
7. The length of a shadow changes from 75 feet to 65 feet.

Practice 4-4**Percent of Change**

Find each percent of change. Describe the percent of change as an increase or decrease. Round to the nearest whole number.

- | | | | |
|--|--|--------------------------------------|--|
| 1. 36 g to 27 g | 2. 40 cm to 100 cm | 3. 90 in. to 45 in. | 4. 500 lb to 1500 lb |
| 5. \$90 to \$84.50 | 6. \$100 to \$140 | 7. \$15 to \$5.50 | 8. 100 mi to 175 mi |
| 9. 280 m to 320 m | 10. 58 to 76 | 11. 60 to 150 | 12. 600 mi to 480 mi |
| 13. 18 to 27 | 14. 290 yd to 261 yd | 15. 26.2 to 22.8 | 16. \$8.50 to \$12.75 |
| 17. $36\frac{1}{2}$ to $29\frac{1}{4}$ | 18. $74\frac{3}{4}$ to $66\frac{1}{2}$ | 19. $6\frac{3}{4}$ to $8\frac{1}{4}$ | 20. $15\frac{1}{2}$ to $18\frac{1}{4}$ |

Find each percent of change. Describe the percent of change as an increase or decrease. Round to the nearest whole number.

21. In 1985, the average price for gasoline was \$1.20/gal. In 2000, the average price for gasoline was \$1.56. Find the percent of change.
22. In 1980, Texas had 27 U.S. Representatives. That number increased to 30 in 2000. Find the percent of change.
23. In 1980, the average annual tuition charge for a four-year public university was \$840. The average annual tuition charge in 2000 was \$3356. What is the percent of change?
24. The United States imported 6,909,000 barrels of oil per day in 1980. In 2000, the United States imported 11,459,000 barrels of oil per day. What is the percent of change?
25. In 1977, the average number of households with cable television was 16.6%. In 2000, the average number of households with cable television was 68%. What is the percent of change?
26. In 1989, there were 38,000 licensed drivers under the age of 16. In 1999, the total number of licensed drivers under 16 was 33,248. Find the percent of change.
27. In 1990, Atlanta, GA, failed to meet air quality standards on 42 days. In 1999, Atlanta failed to meet air quality standards on 61 days. What is the percent of change?

Find the greatest possible error and the percent error for each measurement.

- | | | |
|------------|-------------|------------|
| 28. 3 cm | 29. 0.5 cm | 30. 6 cm |
| 31. 16 in. | 32. 36.85 g | 33. 0.9 cm |

Find the minimum and maximum possible areas for rectangles with the following measurements.

- | | | |
|-------------------------|--------------------------|-----------------------|
| 34. 8 cm \times 10 cm | 35. 3 in. \times 5 in. | 36. 8 m \times 12 m |
|-------------------------|--------------------------|-----------------------|

Find the minimum and maximum possible volume for a rectangular solid with the following measurements.

- | | | |
|--|---|-----------------------------------|
| 37. 16 in. \times 22 in. \times 18 in. | 38. 13 cm \times 15 cm \times 18 cm | 39. 3 m \times 4 m \times 5 m |
|--|---|-----------------------------------|

Reteaching 4-5

Applying Ratios to Probability

OBJECTIVE: Finding probability

MATERIALS: None

The possible results of an experiment are **outcomes**. If you want to find the theoretical probability of a particular event, or a **favorable outcome**, you use this formula.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

Example

Twenty-four students in your homeroom placed lunch orders today. The list sent to the cafeteria is shown at the right. If a student is randomly selected from your class, what is the probability that the student ordered pizza or a hamburger?

Pizza	9
Taco	3
Hot dog	4
Hamburger	6
Tuna sandwich	2

$$\begin{aligned} P(\text{pizza or hamburger}) &= \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} \\ &= \frac{9 (\text{number of pizza orders}) + 6 (\text{number of hamburger orders})}{24 (\text{total number of orders})} \\ &= \frac{15}{24} \\ &= \frac{5}{8} \end{aligned}$$

Exercises

You are fishing in a pond stocked with fish. The table at the right shows a recent fish count. Find each probability.

- $P(\text{sunfish})$
- $P(\text{smallmouth bass})$
- $P(\text{largemouth bass})$
- $P(\text{sunfish or crappie})$
- $P(\text{catfish})$
- $P(\text{not a sunfish})$
- $P(\text{not a crappie or not a sunfish})$
- $P(\text{sunfish or smallmouth bass})$
- $P(\text{catfish or largemouth bass})$
- $P(\text{smallmouth or largemouth bass})$

Sunfish	90
Crappie	33
Smallmouth bass	15
Largemouth bass	12
Total	150

Practice 4-5

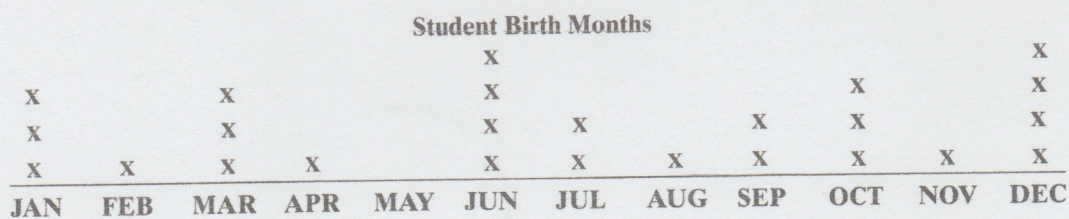
Applying Ratios to Probability

A driver collected data on how long it takes to drive to work.

Time in minutes	20	25	30
Number of trips	4	8	2

- Find P (the trip will take 25 min).
- Find P (the trip will take 20 min).
- Find P (the trip will take at least 25 min).

Use the data in the line plot to find each probability.



- P (June)
- P (October)
- P (first six months of year)
- P (May)
- P (not December)
- P (last three months of year)

A cereal manufacturer selects 100 boxes of cereal at random. Ninety-nine of the boxes are the correct weight. Find each probability.

- P (the cereal box is the correct weight)
- P (the cereal box is not the correct weight)
- There are 24,000 boxes of cereal. Predict how many of the boxes are the correct weight.
- One letter is chosen at random from the word *ALGEBRA*. Find each probability.
 - P (the letter is A)
 - P (the letter is a vowel)
- Patrice has a 40% chance of making a free throw. What is the probability that she will miss the free throw?
- A box of animal crackers contains five hippos, two lions, three zebras, and four elephants. Find the probability if one animal cracker is chosen at random.
 - P (a hippo)
 - P (not an elephant)
 - P (an elephant or a lion)
- Anthony is making a collage for his art class by picking shapes randomly. He has five squares, two triangles, two ovals, and four circles. Find each probability.
 - P (circle is chosen first)
 - P (a square is not chosen first)
 - P (a triangle or a square is chosen first)

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Reteaching 4-6

Probability of Compound Events

OBJECTIVE: Finding the probability of independent and dependent events

MATERIALS: Colored counters and a small bag

To find the probability of two events that are **independent** (the probability of the first **does not** affect the second), multiply the probabilities of the events.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

To find the probability of two events that are **dependent** (the probability of the first **does** affect the second), multiply the probability of the first by the probability of the second happening after the first.

$$P(A \text{ then } B) = P(A) \cdot P(B \text{ after } A)$$

Examples

A bag contains 6 white counters, 5 red counters, and 19 counters of other colors.

- A.** Find the probability of choosing a white and then a red counter if you **replace** the first counter before choosing the second counter.

$$P(A) = P(\text{white}) = \frac{\text{total number}}{\text{total number}} = \frac{6}{30} \text{ or } \frac{1}{5}$$

$$P(B) = P(\text{red}) = \frac{\text{total number}}{\text{total number}} = \frac{5}{30} \text{ or } \frac{1}{6}$$

$$P(A \text{ and } B) = \frac{1}{5} \cdot \frac{1}{6} = \frac{1}{30}$$

The probability of choosing a white and then a red counter (with replacing the first counter) is $\frac{1}{30}$.

- B.** Find the probability of choosing a white and then a red counter if you **do not replace** the first counter before choosing the second counter.

$$P(A) = P(\text{white}) = \frac{\text{total number}}{\text{total number}} = \frac{6}{30} \text{ or } \frac{1}{5}$$

$$P(B) = P(\text{red}) = \frac{\text{total number}}{\text{total number}} = \frac{5}{29}$$

$$P(A \text{ and } B) = \frac{1}{5} \cdot \frac{5}{29} = \frac{1}{29}$$

The probability of choosing a white and then a red counter (without replacing the first counter) is $\frac{1}{29}$.

Exercises

Choose counters of two colors, *A* and *B*. Write down the number of each, and put them in a bag.

- Find the probability of choosing a counter of color *A* and then a counter of color *B* if you replace the first before you pick the second.
- Find the probability of choosing a counter of color *A* and then a counter of color *B* if you do not replace the first pick.

Practice 4-6

Probability of Compound Events

1. Suppose you have a dark closet containing seven blue shirts, five yellow shirts, and eight white shirts. You pick two shirts from the closet. Find each probability.
- | | |
|--|---|
| a. $P(\text{blue then yellow})$ with replacing | b. $P(\text{blue then yellow})$ without replacing |
| c. $P(\text{yellow then yellow})$ with replacing | d. $P(\text{yellow then yellow})$ without replacing |
| e. $P(\text{yellow then white})$ with replacing | f. $P(\text{yellow then white})$ without replacing |
| g. $P(\text{blue then blue})$ with replacing | h. $P(\text{blue then blue})$ without replacing |

A and B are independent events. Find the missing probability.

2. $P(A) = \frac{3}{7}$, $P(A \text{ and } B) = \frac{1}{3}$. Find $P(B)$.
3. $P(B) = \frac{1}{5}$, $P(A \text{ and } B) = \frac{2}{13}$. Find $P(A)$.
4. $P(B) = \frac{15}{16}$, $P(A \text{ and } B) = \frac{3}{4}$. Find $P(A)$.
5. $P(A) = \frac{8}{15}$, $P(B) = \frac{3}{4}$. Find $P(A \text{ and } B)$.
6. Suppose you draw two tennis balls from a bag containing seven pink, four white, three yellow, and two striped balls. Find each probability.
- | | |
|--|---|
| a. $P(\text{yellow then pink})$ with replacing | b. $P(\text{yellow then pink})$ without replacing |
| c. $P(\text{pink then pink})$ with replacing | d. $P(\text{pink then pink})$ without replacing |
| e. $P(\text{striped then striped})$ with replacing | f. $P(\text{striped then striped})$ without replacing |
| g. $P(\text{pink then white})$ with replacing | h. $P(\text{pink then white})$ without replacing |

A and B are independent events. Find the missing probability.

7. $P(A) = \frac{3}{4}$, $P(A \text{ and } B) = \frac{1}{2}$. Find $P(B)$.
8. $P(A) = \frac{3}{7}$, $P(B) = \frac{1}{6}$. Find $P(A \text{ and } B)$.
9. $P(B) = \frac{9}{10}$, $P(A \text{ and } B) = \frac{3}{5}$. Find $P(A)$.
10. $P(B) = \frac{1}{4}$, $P(A \text{ and } B) = \frac{3}{20}$. Find $P(A)$.

Use an equation to solve each problem.

11. A bag contains green and yellow color tiles. You pick two tiles without replacing the first one. The probability that the first tile is yellow is $\frac{3}{5}$. The probability of drawing two yellow tiles is $\frac{12}{35}$. Find the probability that the second tile you pick is yellow.
12. A bag contains red and blue marbles. You pick two marbles without replacing the first one. The probability of drawing a blue and then a red is $\frac{4}{15}$. The probability that your second marble is red if your first marble is blue is $\frac{2}{3}$. Find the probability that the first marble is blue.