**Midsegments of Triangles** 

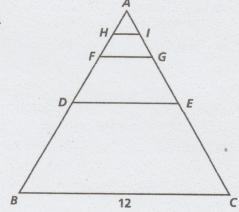
**OBJECTIVE:** Using properties of midsegments to solve problems

MATERIALS: Ruler

## Example

 $\overline{DE}$  is the midsegment of  $\triangle ABC$ .  $\overline{FG}$  is the midsegment of  $\triangle ADE$ .  $\overline{HI}$  is the midsegment of  $\triangle AFG$ . If BC=12, find DE, FG, and HI.

$$DE = \frac{1}{2}BC$$
  $FG = \frac{1}{2}DE$   $HI = \frac{1}{2}FG$   
=  $\frac{1}{2}(12)$  =  $\frac{1}{2}(6)$  =  $\frac{1}{2}(3)$   
= 6 = 3 = 1.5



#### Exercises

Follow the indicated steps to complete each exercise.

- Draw a triangle. Label it  $\triangle XYZ$ .
- Draw the midsegment of  $\triangle XYZ$  parallel to  $\overline{YZ}$ . Label it  $\overline{MN}$ .
- Draw the midsegment of  $\triangle XMN$  parallel to  $\overline{MN}$ . Label it  $\overline{PQ}$ .
- Draw the midsegment of  $\triangle XPQ$  parallel to  $\overline{PQ}$ . Label it  $\overline{RS}$ .
  - **1.** If RS = 4, find the following lengths.

b. MN

c. YZ

- Draw a triangle. Label it  $\triangle PUV$ .
- Draw the midsegment of  $\triangle PUV$  parallel to  $\overline{UV}$ . Label it  $\overline{ST}$ .
- Draw the midsegment of  $\triangle PST$  parallel to  $\overline{ST}$ . Label it  $\overline{QR}$ .
- Draw the midsegment of  $\triangle PQR$  parallel to  $\overline{QR}$ . Label it  $\overline{NO}$ .
  - 2. If QR = 5, find the following lengths.

b. ST

c. UV

- **3.** If NO = 2, find the following lengths.
  - a. QR
- b. ST
- c. UV

**Midsegments of Triangles** 

Use the diagrams at the right to complete the exercises.

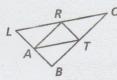
- **1.** In  $\triangle MNO$ , the points C, D, and E are midpoints. CD = 4 cm, CE = 8 cm, and DE = 7 cm.
  - a. Find MO.
- b. Find NO.
- c. Find MN.



- **2.** In quadrilateral WVUT, the points F, E, D, and C are midpoints. WU = 45 in. and TV = 31 in.
  - a. Find CD.
- **b.** Find CF.
- c. Find ED.



- **3.** In  $\triangle LOB$ , the points A, R, and T are midpoints. LB = 19 cm, LO = 35 cm, and OB = 29 cm.
  - a. Find RT.
- b. Find AT.
- c. Find AR.



Find the value of the variable.

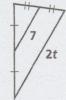
4.



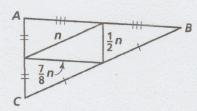
5.



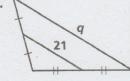
6.



7. Perimeter of  $\triangle ABC = 32 \text{ cm}$ 



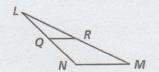
8.



9.

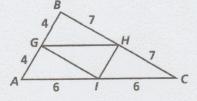


- **10.**  $\overline{QR}$  is a midsegment of  $\triangle LMN$ .
  - a. QR = 9. Find NM.
  - **b.** LN = 12 and LM = 31. Find the perimeter of  $\triangle LMN$ .

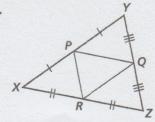


Use the given measures to identify three pairs of parallel segments in each diagram.

11.



12.



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**Bisectors in Triangles** 

**OBJECTIVE:** Determining whether a given point lies on the perpendicular bisector of a segment

MATERIALS: Graph paper

## Example

Given points A(1,3), B(5,1), and C(4,4), does C lie on the perpendicular bisector of  $\overline{AB}$ ?

Plot the points on a coordinate grid. Draw  $\overline{AB}$ . Use the distance formula to determine whether AC = BC.

$$AC = \sqrt{(1-4)^2 + (3-4)^2}$$
  $BC = \sqrt{(5-4)^2 + (1-4)^2}$ 

$$BC = \sqrt{(5-4)^2 + (1-4)^2}$$

$$AC = \sqrt{(-3)^2 + (-1)^2}$$

$$BC = \sqrt{1^2 + (-3)^2}$$

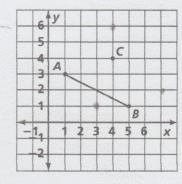
$$AC = \sqrt{9+1}$$

$$BC = \sqrt{1+9}$$

$$AC = \sqrt{10}$$

$$BC = \sqrt{10}$$

Because  $AC = \sqrt{10}$  and  $BC = \sqrt{10}$ , AC = BC, and C lies on the perpendicular bisector of  $\overline{AB}$ .



### Exercises

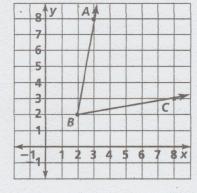
Complete these exercises on bisectors.

- **1.** Given D(3,1), E(7,2), and F(4,5), does F lie on the perpendicular bisector of  $\overline{DE}$ ?
- **2.** Given X(1,2), Y(7,2), and Z(4,6), does Z lie on the perpendicular bisector of  $\overline{XY}$ ?
- **3.** Given H(-4,5), I(-6,2), and J(-1,3), does H lie on the perpendicular bisector of  $\overline{IJ}$ ?
- **4.** Given P(-7, -7), Q(-5, -2), and R(0, -5), does Q lie on the perpendicular bisector of  $\overline{PR}$ ?
- **5.** Point T(-9,5) lies on the perpendicular bisector of  $\overline{UV}$ . If the coordinates of point U are (-2, 1), which of the following are the coordinates of point V?

$$A. (-2,7)$$

**B.** 
$$(-1,6)$$

6. Use the diagram at the right. Which of the following points lies on the angle bisector of  $\angle ABC$ ?

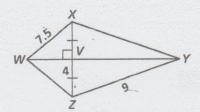


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**Bisectors in Triangles** 

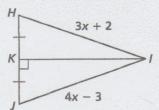
Use the figure at the right for Exercises 1-5.

- **1.** How is  $\overline{WY}$  related to  $\overline{XZ}$ ?
- 2. Find XV.
- 3. Find WZ.
- 4. Find XY.
- **5.** What kind of triangle is  $\triangle WXV$ ?



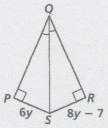
#### Use the figure at the right for Exercises 6-10.

- **6.** Find the value of x.
- 7. Find HI.
- 8. Find JI.
- **9.** If L lies on  $\overline{KI}$ , then L is  $\underline{?}$  from H and J.
- **10.** What kind of triangle is  $\triangle HIJ$ ?



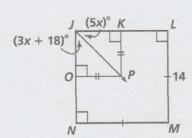
#### Use the figure at the right for Exercises 11-14.

- 11. Find the value of y.
- 12. Find PS.
- 13. Find RS.
- **14.** What kind of triangle is  $\triangle PQS$ ?



### Use the figure at the right for Exercises 15-21.

- **15.** How is  $\overrightarrow{JP}$  related to  $\angle LJN$ ?
- **16.** Find the value of x.
- 17. Find  $m \angle KJP$ .
- **18.** Find  $m \angle OJP$ .
- 19. Find NM.
- **20.** Write a conclusion about point M.
- **21.** What kind of triangle is  $\triangle JOP$ ?



Concurrent Lines, Medians, and Altitudes

**OBJECTIVE:** Finding the point of concurrency of the altitudes of acute, obtuse, and right triangles

MATERIALS: Protractor, straightedge

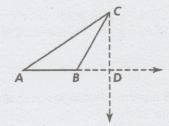
## Example

Draw an obtuse triangle. Find the point of concurrency of the lines containing its altitudes.

Draw obtuse triangle ABC.

Extend side  $\overline{AB}$ .

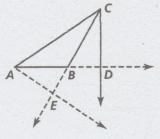
Move the straightedge on your protractor along  $\overrightarrow{AB}$  until C lies directly under 90. Label the point lying directly under C on  $\overrightarrow{AB}$  as point D.



Draw  $\overrightarrow{CD}$ , the ray containing the altitude  $\overline{CD}$ .

Extend side  $\overline{BC}$ .

Move the straightedge on your protractor along  $\overrightarrow{CB}$  until point A lies directly under 90. Label the point lying directly under A on  $\overrightarrow{CB}$  as point E.

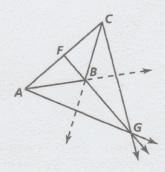


Draw  $\overrightarrow{AE}$ , the ray containing the altitude  $\overline{AE}$ .

Move the straightedge on your protractor along  $\overline{AC}$  until B lies directly under 90. Label the point directly under B on  $\overline{AC}$  as point F.

Draw  $\overrightarrow{FB}$ , the ray containing the altitude  $\overline{BF}$ .

The point of concurrency is G.



#### Exercises

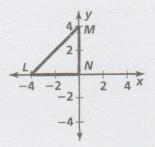
Determine the point of concurrency.

- **1.** Draw an acute triangle. Find the point of concurrency of the lines containing its altitudes.
- **2.** Draw a right triangle. Find the point of concurrency of the lines containing its altitudes.

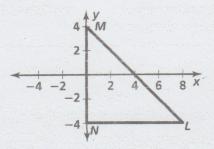
Concurrent Lines, Medians, and Altitudes

Find the center of the circle that circumscribes  $\triangle LMN$ .

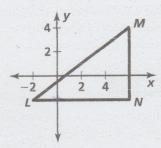
1.



2.



3.

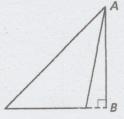


**4.** Construct the angle bisectors for  $\triangle ABC$ . Then use the point of concurrency to construct an inscribed circle.

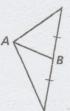
A B

Is  $\overline{AB}$  a perpendicular bisector, an angle bisector, an altitude, a median, or none of these?

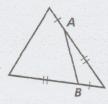
5.



6.



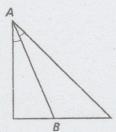
7.



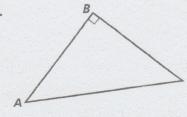
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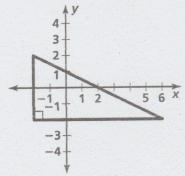


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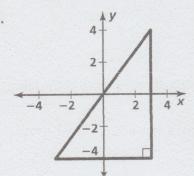


For each triangle, give the coordinates of the point of concurrency of (a) the perpendicular bisectors of the sides and (b) the altitudes.

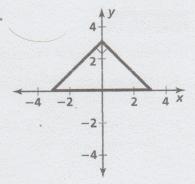
11.



12.



13.



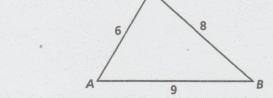
Inequalities in Triangles

**OBJECTIVE:** Using inequalities involving triangle side lengths and angle measures to solve problems MATERIALS: Straightedge

### Example

Use the triangle inequality theorems to answer the questions.

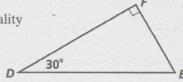
**a.** Which is the largest angle of  $\triangle ABC$ ?  $\overline{AB}$  is the longest side of  $\triangle ABC$ .  $\angle C$  lies opposite  $\overline{AB}$ .  $\angle C$  is the largest angle of  $\triangle ABC$ .



**b.** Which is the shortest side of  $\triangle DEF$ ? Find  $m \angle E$ .

$$m\angle D + m\angle E + m\angle F = 180$$
$$30 + m\angle E + 90 = 180$$
$$120 + m\angle E = 180$$
$$m\angle E = 60$$

Triangle Angle-Sum Theorem Substitution Addition Subtraction Property of Equality



 $\angle D$  is the smallest angle of  $\triangle DEF$ . Because  $\overline{FE}$  lies opposite  $\angle D$ ,  $\overline{FE}$  is the shortest side of  $\triangle DEF$ .

### Exercises

Complete the following exercises.

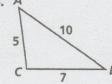
- 1. Draw three triangles, one obtuse, one acute, and one right. Label the vertices. Exchange your triangles with a partner.
  - a. Identify the longest and shortest sides of each triangle.
  - b. Identify the largest and smallest angles of each triangle.
  - c. Describe the relationship between the longest and shortest sides and the largest and smallest angles for each of your partner's triangles.

Which are the largest and smallest angles of each triangle?

2.

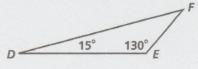




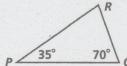


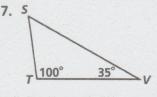
Which are the longest and shortest sides of each triangle?

5.



6.

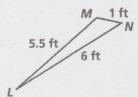


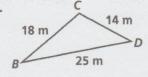


Inequalities in Triangles

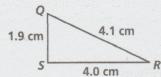
Determine the two largest angles in each triangle.

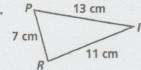
1.

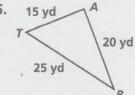


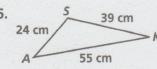


3.









Can a triangle have sides with the given lengths? Explain.

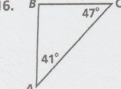
- 7. 4 m, 7 m, and 8 m
- 8. 6 m, 10 m, and 17 m
- 9. 4 in., 4 in., and 4 in.

- 10. 1 yd, 9 yd, and 9 yd
- 11. 11 m, 12 m, and 13 m
- 12. 18 ft, 20 ft, and 40 ft

- 13. 1.2 cm, 2.6 cm, and 4.9 cm
- **14.**  $8\frac{1}{2}$  yd,  $9\frac{1}{4}$  yd, and 18 yd
- 15. 2.5 m, 3.5 m, and 6 m

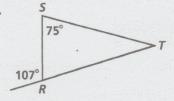
List the sides of each triangle in order from shortest to longest.

16.

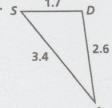


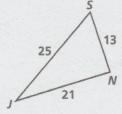


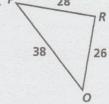
18.



List the angles of each triangle in order from largest to smallest.







The lengths of two sides of a triangle are given. Describe the lengths possible for the third side.

22. 4 in., 7 in.

- 23. 9 cm, 17 cm
- 24. 5 ft, 5 ft

25. 11 m, 20 m

- 26. 6 km, 8 km
- 27. 24 in., 37 in.

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