Patterns and Inductive Reasoning

OBJECTIVE: Using inductive reasoning to make conjectures

MATERIALS: Pennies

Example

Describe the next two figures in the sequence.







Each pile of pennies has two more than the preceding pile, so the next pile will have seven pennies, followed by a pile of nine pennies.

Exercises

Use pennies to model the next two figures in each sequence. Then draw a sketch of the two new figures. Show a maximum of ten coins in one stack.

1.















2.



3.









4.



5



9, 90



6.



Patterns and Inductive Reasoning

Find a pattern for each sequence. Use the pattern to show the next two terms.

- 1. 17, 23, 29, 35, 41, ...
- **2.** 1.01, 1.001, 1.0001, . . .
- **3.** 12, 14, 18, 24, 32, . . .

- **4.** 2, -4, 8, -16, 32, . . .
- **5.** 1, 2, 4, 7, 11, 16, . . .
- **6.** 32, 48, 56, 60, 62, 63, . . .

Name two different ways to continué each pattern.

7. 1, 1, 2, ?

- **8.** 48, 49, 50, ?
- 9. 2, 4, ?

- **10.** A, B, C, ..., Z, ?
- 11. D, E, F, ?
- **12.** A, Z, B, ?

Draw the next figure in each sequence.

13.







?

14.







?

15.



135°



?

Seven people meet and shake hands with one another.

- **16.** How many handshakes occur?
- **17.** Using inductive reasoning, write a formula for the number of handshakes if the number of people is *n*.

The Fibonacci sequence consists of the pattern 1, 1, 2, 3, 5, 8, 13, ...

- 18. What is the ninth term in the pattern?
- **19.** Using your calculator, look at the successive ratios of one term to the next. Make a conjecture.
- **20.** List the first eight terms of the sequence formed by finding the differences of successive terms in the Fibonacci sequence.

Class

Date

Reteaching 1-2

Points, Lines, and Planes

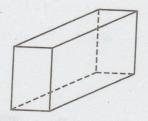
OBJECTIVE: Understanding basic terms and postulates of geometry

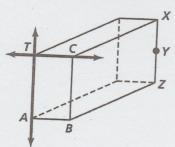
MATERIALS: Colored pencils or markers

Example

Label the figure at the right as indicated.

- a. Label three points that are coplanar as A, B, and C.
- **b.** Label three points that are collinear as X, Y, and Z.
- **c.** Trace two intersecting lines, and label their point of intersection as *T*.

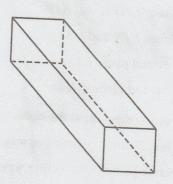




Exercises

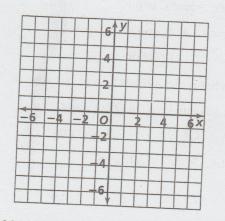
Using colored pencils, label and shade the figure at the right as indicated.

- **1.** With a yellow pencil, shade a plane. Then label three noncollinear points on the plane as *R*, *S*, and *T*.
- **2.** With an orange pencil, shade a plane that intersects the plane you shaded yellow.
- 3. Describe the intersection of the planes you shaded yellow and orange.
- **4.** With a red pencil, label four points that are coplanar as E, F, G, and H.
- **5.** With a blue pencil, label three points that are collinear as P, Q, and R.
- **6.** With a brown pencil, label four points that are not coplanar as W, X, Y, and Z.



Use the grid at the right.

- **7.** Graph the following points on the grid: P(-1, -1), Q(0, 4), R(-3, -5), S(2, 5), and T(3, -4).
- 8. Name three noncollinear points.
- 9. Name three collinear points.
- 10. Name two intersecting lines.



Refer to the diagram at the right for Exercises 1-15.

- 1. Name \overrightarrow{AB} in another way.
- 2. Give two other names for plane Q.
- 3. Why is EBD not an acceptable name for plane Q?

Are the following sets of points collinear?

4. \overrightarrow{AB} and C

5. *B* and *F*

6. \overrightarrow{EB} and A

7. F and plane Q

Are the following sets of points coplanar?

8. E, B, and F

9. \overrightarrow{DB} and \overrightarrow{FC}

10. \overrightarrow{AC} and \overrightarrow{ED}

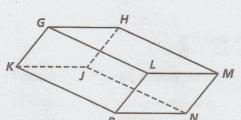
- 11. \overrightarrow{AE} and \overrightarrow{DC}
- 12. F, A, B, and C
- 13. F, A, B, and D
- 14. plane Q and \overrightarrow{EC}
- **15.** \overrightarrow{FB} and \overrightarrow{BD}

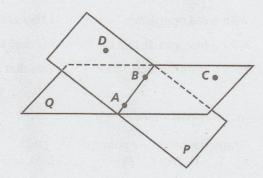
Find the intersection of the following lines and planes in the figure at the right.

- 16. \overrightarrow{GK} and \overrightarrow{LG}
- 17. planes GLM and LPN
- 18. planes GHPN and KJP
- 19. planes HJN and GKL
- **20.** \overrightarrow{KP} and plane KJN
- 21. \overrightarrow{KM} and plane GHL



- 22. Name plane P in another way.
- 23. Name plane Q in another way.
- **24.** What is the intersection of planes P and Q?
- 25. Are A and C collinear?
- 26. Are D, A, B, and C coplanar?
- **27.** Are *D* and *C* collinear?
- **28.** What is the intersection of \overrightarrow{AB} and \overrightarrow{DC} ?
- **29.** Are planes P and Q coplanar?
- **30.** Are \overrightarrow{AB} and plane Q coplanar?
- **31.** Are *B* and *C* collinear?





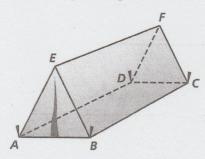
Segments, Rays, Parallel Lines, and Planes

OBJECTIVE: Recognizing parallel lines and parallel planes

MATERIALS: None

Example

Name two pairs of parallel lines.

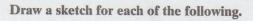


 \overrightarrow{EF} and \overrightarrow{BC} are parallel because they are coplanar lines that do not intersect. So are \overrightarrow{AE} and \overrightarrow{DF} .

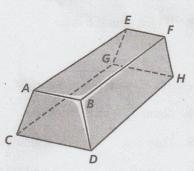
Exercises

For Exercises 1-7, name the lines or planes indicated.

- 1. Name a pair of parallel lines.
- 2. Name a pair of skew lines.
- 3. Name a pair of lines that are neither parallel lines nor skew lines.
- 4. Name a pair of parallel planes.
- 5. Name a pair of planes that intersect in a line.
- 6. Name three planes that intersect at a point.
- 7. Name a pair of skew lines different from the pair named in Exercise 2.



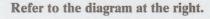
- 8. three parallel lines
- 9. two skew lines
- 10. two intersecting planes
- 11. two parallel planes
- 12. three intersecting lines
- 13. two parallel planes intersected by a line



Segments, Rays, Parallel Lines, and Planes

Write true or false.

- 1. \overrightarrow{XY} is the same as \overrightarrow{YX} .
- 3. If \overrightarrow{AB} and \overrightarrow{AC} are opposite rays, then they are collinear.
- **5.** If the union of two rays is a line, then the rays are opposite rays.



- **7.** Name all segments parallel to \overline{EF} .
- **8.** Name all segments parallel to \overline{FG} .
- 9. Name three pairs of skew lines.



- 10. Which pair(s) of planes is (are) parallel?
- 11. Which pair(s) of planes intersect?
- 12. Which planes intersect in \overrightarrow{MN} ?
- **13.** Which planes intersect in \overrightarrow{RS} ?

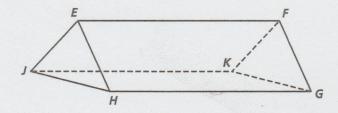
Refer to the diagram at the right.

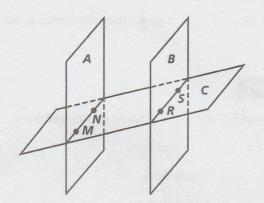
- **14.** Name \overrightarrow{EF} in another way.
- 15. How many different segments can be named?
- **16.** Name a pair of opposite rays with E as an endpoint.
- 17. Name in two different ways the ray opposite \overrightarrow{FG} .
- **18.** Name \overrightarrow{GE} in two other ways.
- **19.** Are \overline{EG} and \overline{GE} the same segment?

Draw each of the following.

- **20.** parallel planes S, T, and U
- **21.** planes R and W intersecting in \overrightarrow{PQ}

- 2. \overrightarrow{XY} is the same as \overrightarrow{YX} .
- **4.** If two rays have the same endpoint, then they form a line.
- **6.** If \overrightarrow{PQ} and \overrightarrow{PR} are the same rays, then Q and R are the same point.





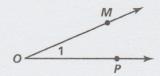


Measuring Segments and Angles

If GJ = 32, find the value of each of the following.

- 1. x
- 2. GH
- 3x x + 16

- 3. HJ
- **4.** Find PD if the coordinate of P is -7 and the coordinate of D is -1.
- **5.** Find SK if the coordinate of S is 17 and the coordinate of K is -5.
- **6.** Find the coordinate of B if AB = 8 and the coordinate of A is -2.
- **7.** Find the coordinate of X if XY = 1 and the coordinate of Y is 0.
- 8. Name the angle at the right in three different ways.



If AX = 45, find the value of each of the following.

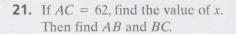
- 9. y
- 10. AQ

 $\begin{array}{c|cccc}
2y+1 & y-1 \\
A & Q & X
\end{array}$

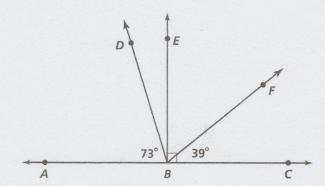
11. QX

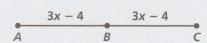
Find the measure of each angle.

- **12.** ∠*EBF*
- **13.** ∠*EBA*
- **14.** ∠*DBE*
- **15.** ∠*DBC*
- **16.** ∠*ABF*
- **17.** ∠*DBF*
- 18. Name all acute angles in the figure.
- 19. Name all obtuse angles in the figure.
- **20.** Name all right angles in the figure.



22. If AC = 206, find the value of x. Then find AB and BC.





The Coordinate Plane

OBJECTIVE: Finding the distance between two points in the coordinate plane

MATERIALS: Graph paper, ruler

Example

Show that the sum of the lengths of the two shortest sides of the triangle is greater than the length of the third side.

Use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$BA = \sqrt{(2 - (-1))^2 + (4 - 2)^2} \qquad AC = \sqrt{(-1 - 4)^2 + (2 - (-1))^2}$$

$$= \sqrt{3^2 + 2^2} \qquad = \sqrt{(-5)^2 + 3^2}$$

$$= \sqrt{9 + 4} \qquad = \sqrt{13} \qquad = \sqrt{34}$$

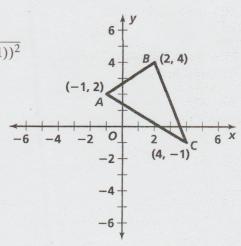
$$BC = \sqrt{(2-4)^2 + (4-(-1))^2}$$

$$= \sqrt{(-2)^2 + 5^2}$$

$$= \sqrt{4+25}$$

$$= \sqrt{29}$$

$$\approx 5.4$$



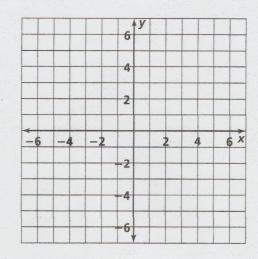
3.6 + 5.4 > 5.8, so BA + BC > AC.

Exercises

≈ 3.6

Use the grid at the right.

- **1.** Graph the coordinates X(-2, 4), Y(6, -3), and Z(2, -2). Connect the vertices to form a triangle.
- **2.** Find the lengths of the sides \overline{XY} , \overline{YZ} , and \overline{XZ} to the nearest tenth.
- **3.** Show that the sum of the lengths of the two shortest sides is greater than the length of the third side.



Find the distance between the points to the nearest tenth.

4.
$$A(-2, -5), B(-4, 7)$$

5.
$$R(3, -4), S(-1, 3)$$

6.
$$G(-4, -5), H(3, 2)$$

7.
$$C(2,5), D(5,-6)$$

8.
$$E(-7,3), F(0,9)$$

9.
$$J(-11, -4), K(-3, -1)$$

10.
$$X(0, 10), Y(-6, -7)$$

11.
$$L(5, -6), M(8, 2)$$

The Coordinate Plane

Graph each point in the coordinate plane.

- 1. A(-2.5)
- **2.** B(5, -2)
- **3.** C(0,6)
- **4.** D(-4,0) **5.** E(-4,-2)

Find the distance between the points to the nearest tenth.

6. L(-4,11), M(-3,4)

7. N(1,0), P(3,8)

8. Q(10, 10), R(10, -2)

9. S(0,5), T(0,-3)

10. U(11,0), V(-1,0)

11. W(2,7), X(1,2)

Find the coordinates of the midpoint of each segment. The coordinates of the endpoints are given.

12. A(6,7), B(4,3)

13. C(-1,5), D(2,-3)

14. E(14, -2), F(7, -8)

- **15.** O(0,0), G(-5,12)
- **16.** H(2.8, 1.1), I(-3.4, 5.7)
- **17.** $J(2\frac{1}{2}, -\frac{1}{4}), K(3\frac{1}{4}, -1)$
- **18.** The midpoint of \overline{AB} is (1,2). The coordinates of A are (-3,6). Find the coordinates of *B*.
- **19.** The midpoint of \overline{CD} is (4, 11). The coordinates of D are (4, 12). Find the coordinates of *C*.
- **20.** The midpoint of \overline{EF} is (-3,7). The coordinates of E are (-3,10). Find the coordinates of F.
- **21.** Graph the points A(2,1), B(2,-5), C(-4,-5), and D(-4,1). Draw the segments connecting A, B, C, and D in order. Are the lengths of the sides of ABCD the same? Explain.
- 22. A crow flies to a point that is 1 mile east and 20 miles south of its starting point. How far does the crow fly?

Quadrilateral PQSR has coordinates as follows: P(0,0), Q(-1,4), R(8,2), and S(7,6).

- 23. Graph quadrilateral POSR.
- **24.** What is the perimeter of *PQSR*?
- **25.** What is the midpoint of \overline{QR} ?

Perimeter, Circumference, and Area

OBJECTIVE: Finding area and perimeter of squares, rectangles, and circles

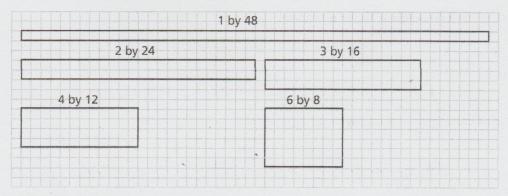
MATERIALS: Graph paper

Example

A rectangle has an area of 48 square units. Its base and height are integers. Use graph paper to determine possible dimensions of the rectangle.

Because the base and height are integers, they must be factors of 48.

List all the factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48. Use the factors to draw possible rectangles on graph paper.



Exercises

Solve for the indicated perimeter or area.

- 1. a. A rectangle has an area of 60 square units. If its base and height are integers, what are its possible dimensions? Sketch each rectangle on graph paper.
 - **b.** Find the perimeter of each rectangle.
- 2. a. A rectangle has an area of 36 square units. If its base and height are integers, what are its possible dimensions? Sketch each rectangle on graph paper.
 - **b.** Which of the possible rectangles has the greatest perimeter?
- 3. Find the dimensions of a rectangle having the least possible perimeter when its base and height are integers and its area is 18 cm².

Draw a circle on graph paper so that the center is at the intersection of grid lines.

- 4. What is the diameter of your circle? What is the radius of your circle?
- 5. Estimate the area of the circle by counting the number of squares and parts of squares in the circle.
- 6. Calculate the area of your circle, using the formula. Round your answer to the nearest tenth.
- 7. How does your calculated result compare with your estimated result?

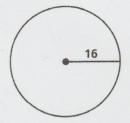
Perimeter, Circumference, and Area

Find the area of each rectangle with the given base and height.

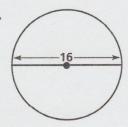
1. base: 3 ft height: 22 in. 2. base: 60 in. height: 1.5 yd 3. base: 2 m height: 120 cm

Find the circumference of each circle in terms of π .

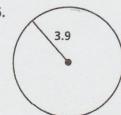
4.



5.



6.



Find the perimeter and area of each rectangle with the given base and height.

7.
$$b = 7 \text{ cm}, h = 6 \text{ cm}$$

8.
$$b = 21 \text{ cm}, h = 2 \text{ cm}$$

9.
$$b = 4$$
 in., $h = 10.5$ in.

10.
$$b = 17$$
 ft, $h = 3$ ft

11.
$$b = 11 \text{ m}, h = 9 \text{ m}$$

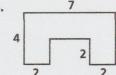
12.
$$b = 13 \text{ m}, h = 7 \text{ m}$$

Find the perimeter and area of each figure. All angles in the figures are right angles.

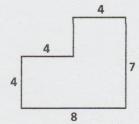
13.



14.



15.

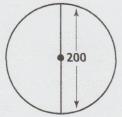


Find the area of each circle in terms of π .

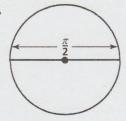
16.



17.



18.



- **19.** Find the area and perimeter of rectangle *ABCD* with vertices A(3,7), B(9,7), C(9,-1), and D(3,-1).
- **20.** Find the perimeter of $\triangle PQR$ with vertices P(-2, 9), Q(7, -3), and R(-2, -3).
- **21.** The circumference of a circle is 26π . Find the diameter and the radius.

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