Relating Graphs to Events

OBJECTIVE: Interpreting and sketching graphs from stories

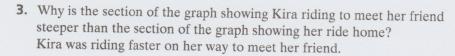
MATERIALS: None

When you draw a graph without actual data, the graph is called a sketch. A sketch gives you an idea of what the graph will look like. Use the description and the sketch to answer the questions.

Example

Kira rides her bike to the park to meet a friend. When she arrives at the park, Kira and her friend sit on the bench and talk for a while. Kira then rides her bike home at a slower pace.

- 1. What does the vertical scale show? It shows distance from home.
- **2.** What does the horizontal scale show? It shows time.

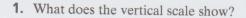


4. Why is the section of the graph flat when Kira is talking to her friend? Kira's distance from home is not changing, but time is still passing.



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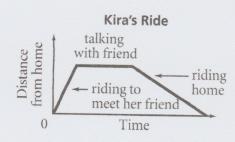
To take photographs of the area where you live for a school project, you ride your bike to the top of Lookout Knoll. The road leading to the top is steep. When you arrive at the top, you rest and take some photographs. On the way back down the same road, you stop to take photographs from another location.



- 2. What does the horizontal scale show?
- **3.** Draw a sketch of the trip comparing the distance you traveled to time. Label the sections.



- **5.** Which part of the graph is steeper, your ride to the top of Lookout Knoll or your ride down? Explain.
- **6.** Suppose the vertical axis represents distance from the base of Lookout Knoll. With all other information remaining the same, draw a sketch of the trip comparing distance from the base of Lookout Knoll to time. Label the sections.



Time

0

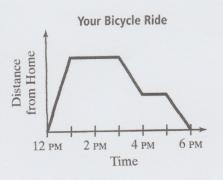
The graph shows the speed a student traveled on the way to school.

- 1. What do the flat parts of the graph represent?
- 2. Circle the sections of the graph that show the speed decreasing.



The graph shows the relationship between time and distance from home.

- 3. What do the flat parts of the graph represent?
- **4.** What do the sections from 3 P.M. to 4 P.M. and from 5 P.M. to 6 P.M. represent?
- 5. What does the section from 12 P.M. to 1 P.M. represent?

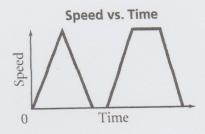


Sketch a graph to describe the following. Explain the activity in each section of the graph.

- 6. your elevation above sea level as you hike in the mountains
- 7. your speed as you travel from home to school
- **8.** the height of an airplane above the ground flying from Dallas, Texas to Atlanta, Georgia
- **9.** the speed of a person driving to the store and having to stop at two stoplights

The graph shows the relationship between time and speed for an airplane.

- 10. Circle the sections of the graph that show the speed increasing.
- 11. Circle the section of the graph that shows the plane not moving.
- **12.** Circle the section of the graph that shows the plane moving at a constant speed.



Relations and Functions

OBJECTIVE: Evaluating functions

MATERIALS: None

A function is a relation that assigns exactly one value in the range to each value in the domain. A function rule may be given as an equation. The function f(x) = 3x + 5 will take a value x and change it into 3x + 5. The function is read "f of x equals three x plus five," not "f times x." Evaluating a function means finding a value in the range for a given value from the domain.

Example

Evaluate f(x) = 4x - 2 for x = 0, 1, and 2.

$$f(x) = 4x - 2$$

$$f(0) = 4(0) - 2$$

$$f(1) = 4(1) - 2$$

$$f(2) = 4(2) - 2$$

$$f(1) = 4(1) - 2$$
 $f(2) = 4(2) - 2$ Substitute each value for x.

$$f(0) = 0 - 2$$

$$f(1) = 4 - 2$$
 $f(2) = 8 - 2$ Simplify.

$$f(2) = 8 - 2$$

$$f(0) = -2$$

$$f(1) = 2$$

$$f(2) = 6$$

Exercises

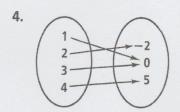
Find the domain and range of each relation.

1.
$$\{(-4,3), (-2,-1), (0,0), (1,4), (2,6)\}$$

2.
$$\{(-6, -4), (-3, -1), (1, 2), (2, 4), (3, 7)\}$$

Determine whether each relation is a function.

3.
$$\{(-1,2),(0,3),(4,3),(0,5)\}$$



Evaluate each function rule for x = -2.

5.
$$f(x) = 4x$$

7.
$$f(x) = x - 2$$

9.
$$f(x) = \frac{1}{2}x + 2$$

6.
$$f(x) = -3x$$

8.
$$f(x) = -2x + 1$$

10.
$$f(x) = -\frac{3}{2}x + 2$$

Find the range of each function, given the domain.

11.
$$g(m) = m^2; \{-2, 0, 2\}$$

13.
$$h(n) = 3n^2 - 2n + 2$$
; $\{-1, 0, 1\}$

15.
$$g(x) = |x| + 2$$
; $\{-4, -2, 4\}$

12.
$$h(x) = -\frac{1}{3}x - 1; \{-3, 0, 6\}$$

14.
$$g(n) = n^2 + n - 2$$
; $\{-2, 0, 2\}$

16.
$$f(x) = -2|x| - 1; \{-3, -2, 3\}$$

Relations and Functions

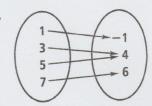
Find the domain and range of each relation.

1.
$$\{(-3, -7), (-1, -3), (0, -1), (2, 3), (4, 7)\}$$

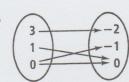
Determine whether each of the following relations is a function.

3.
$$\left\{ (-4, -3), (-2, -2), (0, -1), \left(1, -\frac{1}{2}\right) \right\}$$

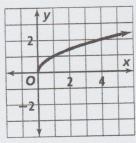
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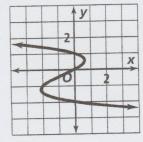
6.



7



8.



Evaluate each function rule for x = 3.

9.
$$f(x) = 2x - 15$$

11.
$$g(x) = \frac{2}{3}x - 1$$

13.
$$h(x) = -0.1x + 2.1$$

10.
$$f(x) = -x + 3$$

12.
$$h(x) = -\frac{1}{2}x - \frac{1}{2}$$

14.
$$g(x) = -\frac{x}{6} + \frac{3}{2}$$

Evaluate each function rule for $x = -\frac{1}{2}$.

15.
$$f(x) = 4x - 2$$

60

17.
$$g(x) = -|x| + 3$$

16.
$$f(x) = -\frac{1}{2}x + 1$$

18.
$$h(x) = x - \frac{1}{2}$$

Find the range of each function for the given domain.

19.
$$f(x) = -3x + 1; \{-2, -1, 0\}$$

20.
$$f(x) = x^2 + x - 2$$
; {-2, 0, 1}

21.
$$h(x) = -x^2; \{-3, -1, 1\}$$

22.
$$g(x) = -\frac{1}{2}|x| + 1; \{-2, -1, 1\}$$

- **23.** For a car traveling at a constant rate of 60 mi/h, the distance traveled is a function of the time traveled.
 - a. Express this relation as a function.
 - **b.** Find the range of the function when the domain is $\{1, 5, 10\}$.
 - c. What do the domain and range represent?

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Function Rules, Tables, and Graphs

OBJECTIVE: Graphing a function

MATERIALS: Graph paper

You can use a rule to model a function with a table and a graph.

Example

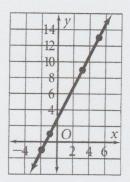
Graph the function y = 2x + 3.

Step 1: Choose four different values for x. Write these values in the first column of the table. Choose some negative values for x.

Step 2: Evaluate the function to find y for each value of x.

X	y = 2x + 3	(x, y)
-2	y = 2(-2) + 3 = -1	(-2, -1)
-1	y = 2(-1) + 3 = 1	(-1,1)
3	y = 2(3) + 3 = 9	(3,9)
5	y = 2(5) + 3 = 13	(5, 13)

Step 3: Plot the ordered pairs to graph the data.



Exercises

Use a table to graph each function. Choose an appropriate number of values for x. Choose some negative values for x.

1.
$$y = 4x + 1$$

3.
$$y = x + 5$$

5.
$$y = x^2 - 4$$

7.
$$y = -|x| + 3$$

2.
$$y = x - 2$$

4.
$$y = |x| - 3$$

6.
$$y = 3x + 3$$

8.
$$y = -x^2 + 4$$

Function Rules, Tables, and Graphs

Model each rule with a table of values and a graph.

1.
$$f(x) = x + 1$$

2.
$$f(x) = 2x$$

3.
$$f(x) = 3x - 2$$

4.
$$f(x) = \frac{3}{2}x - 2$$

5.
$$f(x) = \frac{1}{2}x$$

6.
$$f(x) = -\frac{2}{3}x + 1$$

7.
$$f(x) = x^2 + 1$$

8.
$$f(x) = -x^2 + 2$$

9.
$$f(x) = x - 3$$

- 10. Suppose a van gets 22 mi/gal. The distance traveled D(g) is a function of the gallons of gas used.
 - **a.** Use the rule D(g) = 22g to make a table of values and then a graph.
 - **b.** How far did the van travel if it used 10.5 gallons of gas?
 - c. Should the points of the graph be connected by a line? Explain.
- **11.** The admission to a fairgrounds is \$3.00 per vehicle plus \$.50 per passenger. The total admission is a function of the number of passengers.
 - **a.** Use the rule T(n) = 3 + 0.50n to make a table of values and then a graph.
 - **b**. What is the admission for a car with six people in it?
 - c. Should the points of the graph be connected by a line? Explain.

Graph each function.

12.
$$f(x) = 4x + 2$$

13.
$$f(x) = |-2x|$$

15.
$$f(x) = -|x| - 1$$
 16. $f(x) = 8 - \frac{3}{4}x$

18.
$$f(x) = -\frac{2}{3}x + 6$$

19.
$$f(x) = x^2 - 2x + 1$$

21.
$$y = -x^2 + 1$$

22.
$$y = 9 - x^2$$

14.
$$f(x) = -3x + 7$$

17.
$$f(x) = \frac{2}{3}x - 7$$

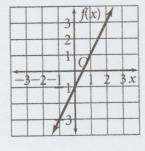
20.
$$f(x) = -\frac{1}{2}x + 3$$

23.
$$y = 2x^2 + x - 2$$

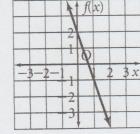
Make a table of values for each graph.



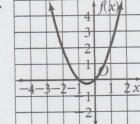
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26.



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Reteaching 5-4

Writing a Function Rule

OBJECTIVE: Writing rules for functions from tables and words

MATERIALS: None

You can write a rule for a function by analyzing a table of values. Look for a pattern in the data table. For each row, ask yourself, "What can I do to the first number to get the second number?" Write the patterns. Circle the pattern that works for all of the data in the table. This is the rule for the function.

Example

х	f(x)
1	3
2	4
3	5

The function rule must be f(x) equals x plus 2. The statement can be written as f(x) = x + 2.

Exercises

Analyze each table and then write the function rule.

1.

х	f(x)
0	0
1 .	3
2	6
3	9

2.

	X	f(x)
Total Parket	0	-1
	1	0
	2	1
CALL TRANSPORT	3	2

3.

х	f(x)
0	0
-1	1
3	9
5	25

Write a function rule for each situation.

- **4.** the length $\ell(w)$ of a box that is two more than four times the width w.
- **5.** the width $w(\ell)$ of a sheet of plywood that is one half the length ℓ .
- **6.** the cost c(a) of a pounds of apples at \$.99 per pound
- 7. the distance d(t) traveled at 65 miles per hour in t hours
- **8.** the value v(q) of a pile of q quarters
- **9.** a worker's earnings e(n) for n hours of work when the worker's hourly wage is \$8.25
- **10.** the distance f(d) traveled in feet when you know the distance d in yards

Writing a Function Rule

Write a function rule for each table.

1.

X	f(x)
0	3
2	5
4	7
6	9

2.

Х	f(x)
0	0
1	3
3	9
5	15

3

X	f(x)
5	0
10	5
15	- 10
20	15

- **4. a.** Write a function rule to calculate the cost of buying bananas at \$.39 a pound.
 - **b.** How much would it cost to buy 3.5 pounds of bananas?
- 5. To rent a cabin, a resort charges \$50 plus \$10 per person.
 - **a.** Write a function rule to calculate the total cost of renting the cabin.
 - **b.** Use your rule to find the total cost for six people to stay in the cabin.

Write a function rule for each table.

6.

х	f(x)
-4	-2
-2	-1
6	3
8	4

7.

Х	f(x)
-3	9
0	0
1	1
5	25

8.

X	f(x)
0	20
2	18
4	16
8	12

- 9. Pens are shipped to the office supply store in boxes of 12 each.
 - **a.** Write a function rule to calculate the total number of pens when you know the number of boxes.
 - **b.** Calculate the total number of pens in 16 boxes.
- **10. a.** Write a function rule to determine the change you would get from a \$20 bill when purchasing items that cost \$1.25 each.
 - **b.** Calculate the change when five of these items are purchased.
 - c. Can you purchase 17 of these items with a \$20 bill?
- 11. You invest \$209 to buy shirts and then sell them for \$9.50 each.
 - a. Write a function rule to determine your profit.
 - **b.** Use your rule to find your profit after selling 24 shirts.
 - c. How many shirts do you need to sell to get back your investment?

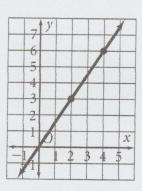
Direct Variation

OBJECTIVE: Using constant of variation to solve problems

MATERIALS: Graph paper and a straight piece of wire or pipe cleaner

Example

Is the equation of the line joining the points (2, 3) and (4, 6) a direct variation? If it is, find the constant of variation.



- Graph the two points. Place the wire on the graph so that it passes through the two points given.

- a. Is the graph a straight line passing through the origin? Yes, it is.
- b. Is the equation of this line a direct variation? Yes, since the line passes through the origin.
- c. What is the constant of variation?

Write the general form of a direct variation.

3 = k(2)

Substitute the coordinates of either point.

The constant of variation is $\frac{3}{2}$.

Exercises

Work with a partner. Draw axes and label them and the origin on your graph paper. One partner places the wire on the graph so that it passes through the two points given. The other partner answers the questions. Exchange roles for each exercise.

Is the equation of the line joining each pair of points a direct variation? If it is a direct variation, what is the constant of variation?

Direct Variation

Is each equation a direct variation? If it is, find the constant of variation.

1.
$$y = 5x$$

2.
$$8x + 2y = 0$$

2.
$$8x + 2y = 0$$
 3. $y = \frac{3}{4}x - 7$

4.
$$y = 2x + 5$$

5.
$$3x - y = 0$$

6.
$$y = \frac{3}{5}x$$

5.
$$3x - y = 0$$
 6. $y = \frac{3}{5}x$ **7.** $-3x + 2y = 0$

$$8. \ -5x + 2y = 9$$

9.
$$8x + 4y = 12$$
 10. $6x - 3y = 0$

10.
$$6x - 3y = 0$$

11.
$$x - 3y = 6$$

12.
$$9x + 5y = 0$$

The ordered pairs in each exercise are for the same direct variation. Find each missing value.

14.
$$(-2, 8)$$
 and $(x, 12)$

16.
$$(x, 8)$$
 and $(6, -16)$

19.
$$(-4,3)$$
 and $(x,6)$

21.
$$\left(\frac{2}{3}, 2\right)$$
 and $(x, 6)$

22.
$$(2.5,5)$$
 and $(x,9)$

24.
$$(9,3)$$
 and $(x,-2)$

For the data in each table, tell whether y varies directly with x. If it does, write an equation for the direct variation.

25.

-
У
8
14
20

26.

	Х	У
-	-3	-2
	3	2
	9	6

27.

X	У
4	3
5	4.5
11	13.5

28.

-	X	У
Contraction	-2	-2.8
-	3	4.2
-	8	11.2

- 29. Charles's Law states that at constant pressure, the volume of a fixed amount of gas varies directly with its temperature measured in degrees Kelvin. A gas has a volume of 250 mL at 300° K.
 - a. Write an equation for the relationship between volume and temperature.
 - **b.** What is the volume if the temperature increases to 420° K?
- 30. Your percent grade varies directly with the number of correct answers. You got a grade of 80 when you had 20 correct answers.
 - a. Write an equation for the relationship between percent grade and number of correct answers.
 - b. What would your percent grade be with 24 correct answers?
- 31. The amount of simple interest earned in a savings account varies directly with the amount of money in the savings account. You have \$1000 in your savings account and earn \$50 in simple interest. How much interest would you earn if you had \$1500 in your savings account?

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Describing Number Patterns

OBJECTIVE: Finding the common difference and writing the next several terms in a sequence

MATERIALS: None

When trying to determine the common difference of an arithmetic sequence or find the pattern, it is helpful to attempt to express each term in the sequence as an expression involving the same number.

Example

Find the common difference of the sequence: 4, 7, 10, 13, ...

Let n = the term number in the sequence.

Let A(n) = the value of the nth term of the sequence.

$$A(1) = 4$$

$$A(2) = 7 = 4 + 1(3)$$

← 3 is the common difference.

$$A(3) = 10 = 4 + 6 = 4 + 2(3)$$

← Notice that the 3 in each expression is multiplied by a number one less than the term number.

$$A(4) = 13 = 4 + 9 = 4 + 3(3)$$

$$A(n) = 4 + 3 + 3 + \dots + 3 = 4 + (n-1)3$$

The formula for the sequence is A(n) = 4 + (n - 1)3.

You could use the formula for the sequence to determine the next several terms simply by substituting a specific term in for n. For example:

5th term
$$A(5) = 4 + (5 - 4)$$

5th term
$$A(5) = 4 + (5 - 1)3$$

 $A(5) = 16$ 100th term $A(100) = 4 + (100 - 1)3$
 $A(100) = 301$

Exercises

Find the common difference of each sequence.

Find the next two terms in each sequence.

Find a formula for the sequence in the exercise indicated and use it to determine the fifth and tenth terms of the sequence.

7. Exercise 1

8. Exercise 2

9. Exercise 3

Describing Number Patterns

Find the common difference of each arithmetic sequence.

- 1. 10, 16, 22, 28, ...
- **3.** -12, -17, -22, -27, ...
- 5. $4, 4\frac{1}{2}, 5, 5\frac{1}{2}, \dots$
- **7.** 9, 10.5, 12, 13.5, ...
- 9. 8, 9.1, 10.2, 11.3, ...
- 11. -3, -0.6, 1.8, 4.2, ...
- 7,100,12,100,100
- Find the next two terms in each sequence.

- **15.** 1, -4, -9, -14, . . .
- 17. 2.7. 4. 5.3, 6.6, ...
- **19.** $6\frac{1}{3}$, $4\frac{2}{3}$, 3, $1\frac{1}{3}$, ...

- 2. 9, 6, 3, 0, ...
- **4.** -11, -8, -5, -2, ...
- **6.** $7\frac{1}{2}$, 7, $6\frac{1}{2}$, 6, . . .
- **8.** 1, -1.5, -4, -6.5, . . .
- **10.** -9, -8.1, -7.2, -6.3, . . .
- **12.** 6.2, 4.5, 2.8, 1.1, . . .

- **16.** $\frac{1}{2}$, $-\frac{1}{2}$, $-\frac{3}{2}$, $-\frac{5}{2}$, ...
- **18.** 9.8, 0.7, -8.4, -17.5, . . .
- **20.** $2\frac{1}{2}, \frac{3}{4}, -1, -2\frac{3}{4}, \dots$

Find the fifth, tenth, and hundredth terms of each sequence.

25.
$$\frac{1}{4}$$
, $-\frac{1}{4}$, $-\frac{3}{4}$, $-\frac{5}{4}$, ...

30.
$$-3\frac{1}{2}$$
, $-3\frac{3}{4}$, -4 , $-4\frac{1}{4}$, ...

Determine whether each sequence is arithmetic. Justify your answer.

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- 35. Renting a backhoe costs a flat fee of \$65 plus an additional \$35 per hour.
 - **a.** Write the first four terms of a sequence that represents the total cost of renting the backhoe for 1, 2, 3, and 4 hours.
 - b. What is the common difference?
 - c. What are the 5th, 24th, 48th, and 72nd terms in the sequence?